A DOA Estimation Algorithm Exploiting Multiple FM Channels for Passive Bistatic Radar

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Abstract

An FM based passive bistatic radar (PBR) is a passive radar technique exploiting the multiple FM transmitters. In such PBR systems, the direction-of-arrival (DOA) for target localization is considered as one of the important parameters to be estimated. The conventional methods have mainly focused on the single FM channel configuration for DOA estimation. Recently, the multi-channel based DOA estimation schemes have been developed. However, they have not been fully secured due to the lack of the theoretical analysis. Since the maximum likelihood (ML) technique is known to be approximately the minimum variance unbiased (MVU) estimator, thus, we develop an ML angle estimator exploiting the multiple FM channels. From the simulation results, we show that the proposed method asymptotically approaches the Cramer-Rao bound (CRB) and can be established without cumbersome process of the conventional method.

Keywords: Passive bistatic radar (PBR), direction-of-arrival estimation (DOA), FM broadcasting, maximum likelihood (ML), Cramer-Rao bound (CRB)

1. Introduction

Passive bistatic radar (PBR) is a passive radar technique exploiting the communication and broadcasting systems [1-3]. Among the various transmission signals, FM radio signal has been very popular for PBR detection because it has suitable transmit power level for detecting targets compared to other illuminators [4].

In order to estimate the target location, the direction-of-arrival (DOA) estimates of the target echoes can be utilized [5]. Several research articles have focused on the DOA estimation for PBR system, however, the articles have considered only for the single FM channel configuration because of its simplicity [6, 7].

With the development of the computational speed of the signal processing hardware, the multi-channel configuration of PBR has been recently studied. For the DOA estimation based on the multi-channel configuration, a weighted sum of the DOA values estimated from each single FM channel was developed in [8] and [9]. The weighted sum of the DOA estimates can be calculated by

$$\hat{\theta} = \frac{1}{N_{ch}} \sum_{k=1}^{N_{ch}} \alpha_k \hat{\theta}_k, \tag{1}$$

where $\hat{\theta}$ denotes a weighted sum of $\hat{\theta}_k$, N_{ch} represents the number of FM channels, α_k is defined as the element of the weight vector for each channel and $\hat{\theta}_k$ means the DOA estimate of *k*th channel. By presenting the estimation accuracy of the single and the multi-channel case, the weighted sum of the DOA estimates was shown to have lower error than the estimation error of the single-channel case [8, 9]. However, it is difficult to determine the proper value of α_k. Furthermore, the method presented in [8] and [9] has not been secured due to the lack of the theoretical analysis.

In this paper, we propose a maximum likelihood (ML) angle estimator for the multi-channel FM based PBR system. Since the ML technique is known to be approximately the minimum variance unbiased (MVU) estimator [8, 9], the proposed method has its theoretical basis. Furthermore, the proposed method can calculate the DOA estimate without cumbersome process for determining the weight vector as in [8, 9]. By comparing the root-mean-square error (RMSE) of the proposed method with the CRB, we show that the RMSE of the ML angle estimator approaches the CRB.

2. Signal model

Fig. 1 shows the concept of the multi-static configuration of PBR system. By obtaining the target echo signals caused by the multiple FM transmitters, the PBR receiver can estimate the target location and velocity. For the DOA estimation, we assume that N_{ch} target echoes are received from the antenna array

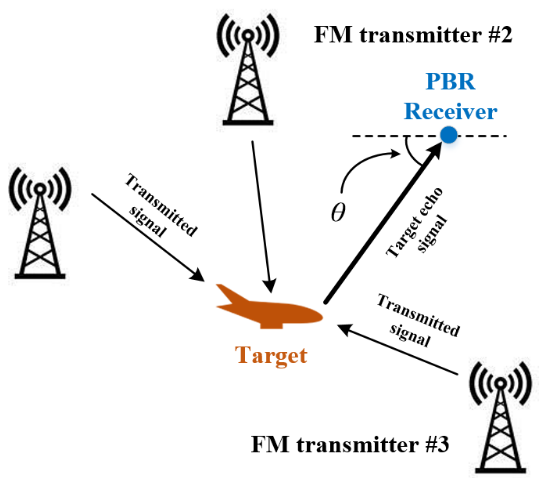

Figure 1. The multi-static geometry of the PBR system.

with M elements. Since one characteristic of the FM broadcasting is multi-frequency network (MFN), each target echo signal has its own carrier frequency if those target echo signals are caused by a specific target. Furthermore, the signal-to-noise ratio (SNR) of the direct-path signals, exploited as the reference signal of matched filter, is assume to be high enough to cancel the clutter. In addition, the time and frequency difference between the direct-path and target echo are assumed to be known.

Then, the received signal at qth channel (q=1,…,N_{ch}) can be modeled as

$$\mathbf{x}^{(q)}(t_n) = \mathbf{A}^{(q)}(\mathbf{\Theta})\mathbf{s}^{(q)}(t_n) + \mathbf{w}^{(q)}(t_n), \quad n = 1, \ldots, N, \tag{2}$$

where N represents the number of observation samples, $\mathbf{A}^{(q)}(t_n)$ is the array manifold matrix, $\mathbf{w}^{(q)}(t_n)$ denotes a white Gaussian noise process vector with M elements, and $\mathbf{s}^{(q)}(t_n)$ represents the target echo signal vector with K elements. $\mathbf{\Theta} = [\theta_1, \ldots, \theta_K]^T$ is defined as a vector of DOAs of K targets. kth element of $\mathbf{s}^{(q)}(t_n)$, $s_k^{(q)}(t_n)$ can be written as

$$s_k^{(q)}(t_n) = \gamma_k^{(q)} y_k^{(q)}(t_n), \quad k = 1, \ldots, K, \tag{3}$$

where $\gamma_k^{(q)}$ represents the complex-valued amplitude, and $y_k^{(q)}(t_n)$ denotes the target echo signal with normalized power. Thus, the target echo signal is a time-delayed and frequency-shifted version of the transmitted FM signal.

The signal model in (2) can be reduced to the simple matrix form. When we define the matrices without the superscript (q) as the block diagonal matrix according to q, then (2) can be rewritten as

$$\begin{aligned}\mathbf{x}(t_n) &= \mathbf{A}(\mathbf{\Theta})\mathbf{s}(t_n) + \mathbf{w}(t_n) \\ &= \mathbf{D}(\mathbf{\Theta})\mathbf{y}(t_n) + \mathbf{w}(t_n),\end{aligned} \tag{3}$$

where

$$\mathbf{D}(\mathbf{\Theta}) = \mathrm{blkdiag}\{\mathbf{D}^{(1)}(\mathbf{\Theta}), \ldots, \mathbf{D}^{(N_{ch})}(\mathbf{\Theta})\}, \tag{4}$$

$$\mathbf{D}^{(q)}(\mathbf{\Theta}) = \mathbf{A}^{(q)}(\mathbf{\Theta})\mathbf{\Gamma}^{(q)}, \tag{5}$$

and

$$\mathbf{\Gamma}^{(q)} = \mathrm{diag}\{\gamma_1^{(q)}, \ldots, \gamma_K^{(q)}\}. \tag{6}$$

According to the assumptions defined in this paper, the unknown parameters in this signal model are $\mathbf{\Theta}$ and $\gamma_k^{(q)}$.

3. ML angle estimator exploiting the multiple FM channels

The log-likelihood function is derived as [10]

$$-\ln|\mathbf{W}| - \mathrm{tr}\left\{\mathbf{W}^{-1}\frac{1}{N}\sum_{n=1}^{N}\mathbf{e}(t_n)\mathbf{e}^H(t_n)\right\}, \tag{7}$$

where $\mathbf{e}(t_n) = \mathbf{x}(t_n) - \mathbf{D}(\mathbf{\Theta})\mathbf{y}(t_n)$, $\mathrm{tr}\{\cdot\}$ is defined to be the sum of the elements on the main diagonal, and $\mathbf{W}$ represents the covariance matrix of the noise vector. Since $\mathbf{x}(t_n)$ and $\mathbf{y}(t_n)$ are known, the array manifold matrix $\hat{\mathbf{D}}(\mathbf{\Theta})$ can be estimated by minimizing the following equation

$$J = \left|\frac{1}{N}\sum_{n=1}^{N}\mathbf{e}(t_n)\mathbf{e}^H(t_n)\right|. \tag{8}$$

Let

$$\hat{\mathbf{R}}_{yx} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}(t_n)\mathbf{x}^H(t_n), \tag{9}$$

then $\hat{\mathbf{R}}_{xx}$ and $\hat{\mathbf{R}}_{yy}$ can be similarly defined as in (9). Using the covariance matrices of $\mathbf{y}(t_n)$ and $\mathbf{x}(t_n)$, (8) can be rewritten as [10]

$$J = \left|\left[\mathbf{D}(\mathbf{\Theta}) - \hat{\mathbf{R}}_{yx}^H\hat{\mathbf{R}}_{yy}^{-1}\right]\hat{\mathbf{R}}_{yy}\left[\mathbf{D}(\mathbf{\Theta}) - \hat{\mathbf{R}}_{yx}^H\hat{\mathbf{R}}_{yy}^{-1}\right]^H + \hat{\mathbf{W}}\right|, \tag{10}$$

where

$$\hat{\mathbf{W}} = \hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H\hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{R}}_{yx}. \tag{11}$$

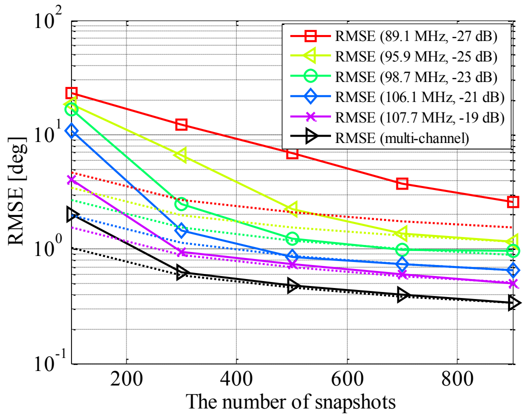

Figure 3. The RMSE of the single-channel and the multi-channel based PBR according to the number of snapshots.

To minimize the cost function in (10), the estimate of the array manifold matrix can be obtained by

$$\hat{\mathbf{D}}(\boldsymbol{\Theta}) = \hat{\mathbf{R}}_{yx}^{H}\hat{\mathbf{R}}_{yy}^{-1}. \tag{12}$$

By using the decoupling method in [10], the cost function in (10) can be separated into the cost function for each θ_k. The cost function of θ_k, J_k, can be written as

$$J_k = \mathrm{tr}\left\{\left[\mathbf{F}(\theta_k)-\hat{\mathbf{F}}_k\right]^H \hat{\mathbf{W}}^{-1}\left[\mathbf{F}(\theta_k)-\hat{\mathbf{F}}_k\right]\right\}, \tag{13}$$

where

$$\mathbf{F}(\theta) = \mathrm{blkdiag}\{\mathbf{d}^{(1)}(\theta), ..., \mathbf{d}^{(N_{ch})}(\theta)\}, \tag{14}$$

and

$$\hat{\mathbf{F}}_k = \mathrm{blkdiag}\{\hat{\mathbf{d}}^{(1)}(\theta_k), ..., \hat{\mathbf{d}}^{(N_{ch})}(\theta_k)\}. \tag{15}$$

$\mathbf{F}(\theta)$ represents the array manifold matrix of the incident angle θ for all FM channels. $\hat{\mathbf{F}}_k$ is defined as the estimate of $\mathbf{F}(\theta)$ and be obtained from (12). In (14) and (15), $\mathbf{d}(\theta)$ represents the steering vector of incident angle θ. Thus, by searching in θ and amplitude components in (13), the unknown parameters can be obtained. However, there is a problem that the multi-dimensional search for unknown parameters is still required.

To solve this problem, the partial derivative of each unknown parameters can be used. By deriving the partial derivative of θ_k and $\hat{\gamma}_k^{(q)}$, then we have

$$\hat{\theta}_k = \arg\max_{\theta} \mathrm{tr}\left\{\frac{\mathbf{Z}^H(\theta)\hat{\mathbf{W}}^{-1}\hat{\mathbf{F}}_k}{\mathbf{Z}^H(\theta)\hat{\mathbf{W}}^{-1}\mathbf{Z}^H(\theta)}\right\} \tag{16}$$

and

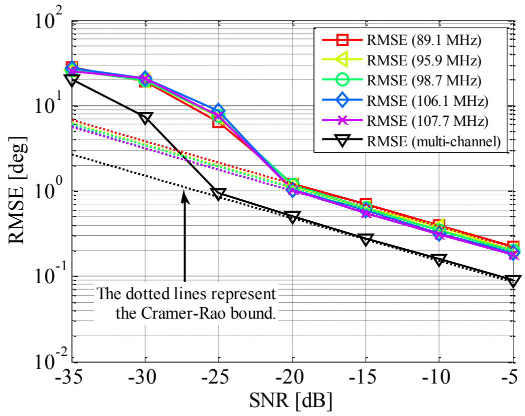

Figure 2. The RMSE of the single-channel and the multi-channel based PBR according to the SNR of target echo signal.

$$\hat{\gamma}_k^{(q)} = \frac{\mathbf{a}^{(q)H}(\hat{\theta}_k)[\hat{\mathbf{W}}^{(q)}]^{-1}\hat{\mathbf{d}}_k^{(q)}}{\mathbf{a}^{(q)H}(\hat{\theta}_k)[\hat{\mathbf{W}}^{(q)}]^{-1}\mathbf{a}_k^{(q)}(\hat{\theta}_k)}, \tag{17}$$

where $\mathbf{Z}(\theta) = \mathbf{\Gamma}_k^{-1}\mathbf{F}(\theta)$. The unknown parameters can be obtained from the (16) and (17).

4. Simulation results

We present the RMSE of the DOAs of the target. For all simulations, we consider the uniform linear array with M = 9 elements, and the inter-sensor distance is 1.393 m. The observation time is 1 ms. A set of carrier frequencies for the multi-channel ML is {89.1, 95.9, 98.7, 106.1, 107.7} MHz and we assume that 5 FM channels are used to estimate the target direction. The incident angle of the target echo signal is set to -15°. The number of ensembles is 500.

In Fig. 2, the RMSE of the ML angle estimator exploiting the multiple channels according to the number of snapshots is derived. The dotted lines in Figs. 2 and 3 represent the CRBs. The SNR for each target echo signal is set to {-27, -25, -23, -21, -19} dB. As shown in Fig. 2, the RMSE of the proposed method approaches the CRB as the number of snapshots increases. Since the multi-channel based ML exploits all target echo signals with various SNR values, the RMSE of the proposed method has lower values than that of the single-channel based ML technique.

In Fig. 3, the RMSE of the single-channel and the multi-channel ML according to the SNR is compared. As shown, the RMSE of the single-channel is lower than the multi-channel case. The RMSE of the multi-channel based ML approaches CRB at the lower SNR value than that of the single-channel case. After approaching the CRB, the RMSE of the multi-channel ML also has lower value than that of the single-channel case. The difference between the single-channel and the multi-channel case is shown to be approximately 6 dB.

Figs. 2 and 3 show that the proposed ML method can achieve more accurate DOA estimation performance by exploiting the multiple FM channels. Furthermore, comparing with the RMSE of the single-channel exploitation, the RMSE of the proposed method approaches the CRB at the lower SNR and the smaller number of snapshots.

5. Conclusions

We derived the multi-channel based ML technique for the DOA estimation in the application of an FM radio based PCL system. By exploiting the property of the multi-channel configuration of the FM broadcasting, we showed that the estimation accuracy can be improved from the multiple FM channels. Furthermore, we also found that the RMSE of the proposed method asymptotically approaches the CRB.

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