CRLB based Performance Analysis of Target Localization in a PCL system

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Abstract

This paper analyzes the localization performance of a target in a PCL (passive coherent location) system, where several conditions should be considered to derive the estimation performance concerning the actual operating situation. First, SNR (signal-to-noise ratio) of the received signal according to the target location should be considered. Second, the bistatic range error according to the SNR of the received signal should be taken into account. Finally, bistatic range error is an important factor to determine the estimation performance of the target location. Involving the above three conditions, we derive Cramer-Rao lower Bound corresponding to each condition and then carry out the performance analysis concerning the actual operation.

Keywords: Passive Coherent Location (PCL), Target Localization, Cramer-Rao Lower Bound (CRLB)

1. Introduction

A passive coherent location (PCL) system is a passive radar technique for detecting a target and estimating the target location by using the commercial broadcast signals such as FM (frequency-modulation), DVB (digital video broadcasting), and DMB (digital multimedia broadcasting) [1-3]. The PCL system can estimate the location of a target from the bistatic range information, which is defined as the time difference between the direct path signal and the target echo signal. The bistatic range information geometrically corresponds to the ellipse with the focal points of which are the location of the transmitter and the receiver as shown in Fig. 1. The position of the target can be estimated from the intersection of multiple ellipses using several pairs of bistatic radar geometry as shown in Fig. 2 [4].

In the PCL system, there are three variables to be considered to derive the localization performance of the target: the location of the target, the SNR (signal-to-noise ratio) of the target echo signal, and the estimation error of the bistatic range. The SNR of the target echo signal varies with the location of the target [5]. The estimation error of the bistatic range can be calculated as a function of the SNR of the received signal [6]. The bistatic range error affects the estimation performance of the target location [6]. From these three elements, the variance of the target localization error according to the target location can be derived.

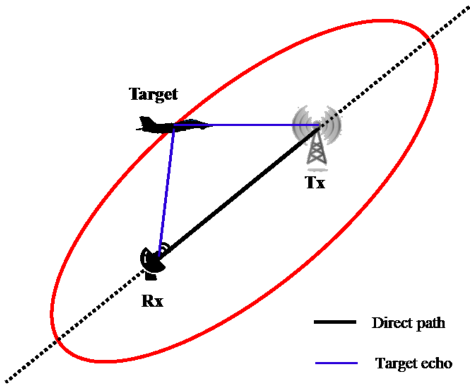

Figure 1. Bistatic range ellipse.

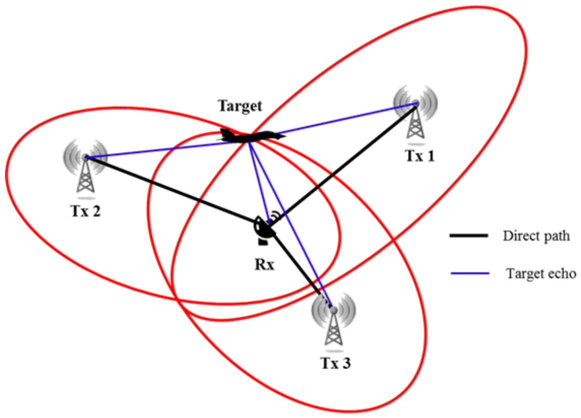

Figure 2. Multiple bistatic range ellipses using multiple transmitters.

In this paper, we calculate the SNR of the target echo signal at each location from the bistatic radar equation in the three-dimensional space and derive Cramer-Rao lower bound (CRLB) of the bistatic range error corresponding to the SNR of each location. Finally, the theoretical estimation performance of the target location is analyzed using the CRLB with three bistatic range error values in three transmitters and one receiver.

2. Steps for estimation performance analysis of the target location

The following three steps are required to derive the estimation performance of the target location.

1. SNR calculation of the target echo signal
2. CRLB of the estimation error of bistatic range
3. CRLB of the estimation error of the target location

In each step, the formulas are defined in order to analyze the estimation performance of the target location.

1. SNR calculation of the target echo signal [5]

The SNR of the target echo signal can be derived from the bistatic radar equation. If the power of the received signal and the power of the noise are defined as P_r, and P_n, respectively, then the SNR of the target echo signal is written as

$$SNR = \frac{P_r}{P_n} = P_t G_t \cdot \frac{1}{(4\pi r_1 r_2)^2} \cdot \sigma_b \cdot \frac{G_r \lambda^2}{4\pi} \cdot L \cdot \frac{1}{kT_0 BF}, \quad (1)$$

where P_t represents the transmit power of the illuminator, G_t is the transmit antenna gain, r_1 is the transmitter-to-target range, r_2 is the target-to-receiver range, σ_b is the target bistatic RCS (radar cross-section), G_r is the receive antenna gain, λ is the signal wavelength, L is system loss, k is the Boltzmann's constant, T_0 is the noise reference temperature (290 K), B is the receiver effective bandwidth, and F is the receiver effective noise figure.

2. CRLB of the estimation error of bistatic range [6]

The CRLB of the bistatic range error according to the target location can be derived from the SNR, signal bandwidth, and signal acquisition time. The variance of the bistatic range error is represented by

$$\mathrm{var}(\hat{R}) \geq \frac{c^2/4}{2 \cdot B \cdot T \cdot SNR \cdot \bar{F}^2}, \quad (2)$$

where T is signal acquisition time, c is velocity of light, $\bar{F}^2$ is mean square bandwidth of the signal [6]. Fig. 3 shows the bistatic range error according to the SNR when B = 100 kHz and T = 1 s. It can be seen that as the SNR increases 10 dB, the bistatic range error decreases 10 times. The calculated bistatic distance error can be used to derive the estimation performance of the target location.

3. CRLB of the estimation error of the target location [6]

The CRLB of the bistatic range is used to derive the estimation performance of the target location If the bistatic range at the i-th transmitter is denoted by R_i, and the variance of the bistatic range is σ_i^2, then the average **R** and covariance **C** of the measurement vectors according to the target location $\mathbf{x}_t = [x_t\ y_t\ z_t]$ are defined as follows

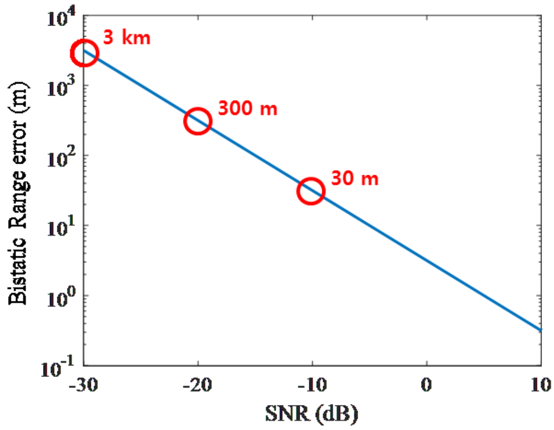

Figure 3. Bistatic range error according to SNR.

$$\mathbf{R}(\mathbf{x}_t) = [R_1(\mathbf{x}_t)\quad R_2(\mathbf{x}_t)\quad R_3(\mathbf{x}_t)], \quad (3)$$

$$\mathbf{C}(\mathbf{x}_t) = \mathrm{diag}([\sigma_1^2(\mathbf{x}_t)\quad \sigma_2^2(\mathbf{x}_t)\quad \sigma_3^2(\mathbf{x}_t)]). \quad (4)$$

The Fisher information matrix (FIM) can be obtained by using the mean and covariance of the measurement vectors. The (i, j) components of the FIM are given as

$$[\mathbf{I}(\mathbf{x}_t)]_{ij} = \frac{\partial \mathbf{R}(\mathbf{x}_t)}{\partial [\mathbf{x}_t]_i} \mathbf{C}^{-1}(\mathbf{x}_t) \frac{\partial \mathbf{R}(\mathbf{x}_t)}{\partial [\mathbf{x}_t]_j}. \quad (5)$$

The FIM defined in the three-dimensional space is a 3 by 3 matrix, and the inverse of FIM defined in Eq. (5) becomes the CRLB matrix of the estimation performance of the target location. The CRLB matrix can be obtained by

$$\mathbf{C}_{CRLB}(\mathbf{x}_t) = [\mathbf{I}(\mathbf{x}_t)]^{-1} = \begin{bmatrix} \sigma^2_{x,CRLB} & \sigma_{xy,CRLB} & \sigma_{xz,CRLB} \\ \sigma_{xy,CRLB} & \sigma^2_{y,CRLB} & \sigma_{yz,CRLB} \\ \sigma_{xz,CRLB} & \sigma_{yz,CRLB} & \sigma^2_{z,CRLB} \end{bmatrix}. \quad (6)$$

Finally, the diagonal component of CRLB derived from Eq. (6) represents the estimated performance error of the x-axis, y-axis and z-axis of the coordinate plane.

3. Simulation result

We present the estimation performance of target location according to the position of a target. In the following simulations, three transmitters are located at x_{t1} = [0 50 0.15] km, x_{t2} = [-25√3 -25 0.1] km, and x_{t3} = [25√3 -25 0.3] km. The receiver location is x_r = [0 0 0] km as shown in Fig 4. The parameters used in Eq. (1) and (2) are summarized in Table 1, and the parameter reference is based on the FM signal [5]. The target is located in the x-axis and y-axis -200 ~ 200 km, and the altitude of the target is fixed at 10 km.

Figs. 5 and 6 show the *x*-axis estimation error and the *y*-axis estimation error of the target, respectively. When the target is located within a radius of 100 km, the estimation error of the *x*-axis and *y*-axis occurs about 200 m. When the target is located within a radius of 200 km, the estimation error of the *x*-axis and *y*-axis occurs about 4 km.

Table 1 . Simulation parameters

P_tG_t	250 kW
σ_b	100 m^2
G_r	9.03 dBi
λ	3.2
L	- 5 dB
B	100 kHz
F	20 dB
T	1 s

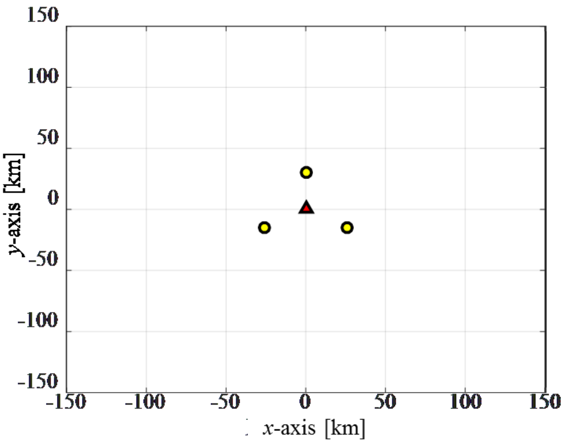

Figure 4. The placement of three transmitters and one receiver.

Fig. 7 shows the *z*-axis estimation error of the target. When the target is located within a radius of 100 km, the estimation error of the *z*-axis occurs about 1.6 km, and the target is located within a radius of 200 km, the estimation error of the *z*-axis occurs about 80 km. As a result, it can be seen that the altitude estimation performance is not better than the horizontal estimation performance.

4. Conclusion

We analyzed the estimation performance of the target location through three steps for the analysis considering the actual operating environment in the PCL system. The theoretical lower bound was obtained by using the CRLB of each step, and the theoretical estimation performance of the target location was derived using the CRLB. From the analysis result, we have found that the altitude estimation performance is not better than the horizontal estimation performance. When the target is separated by 100 km from the receiver, a horizontal error of 200 m occurs and an altitude error of 1.6 km occurs, and the target is separated by 200 km from the receiver, a horizontal error of 4 km occurs and an altitude error of 80 km occurs.

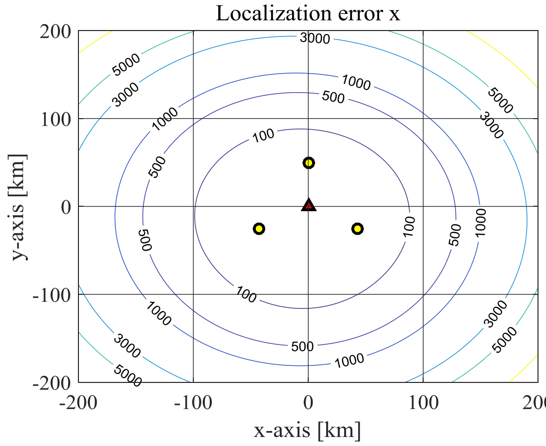

Figure 5. Localization error of *x*-axis according to target location.

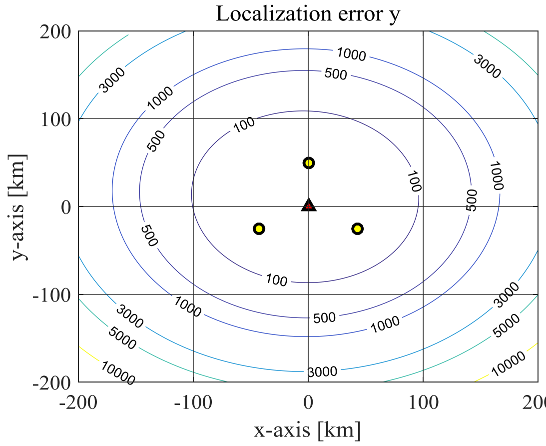

Figure 6. Localization error of *y*-axis according to target location.

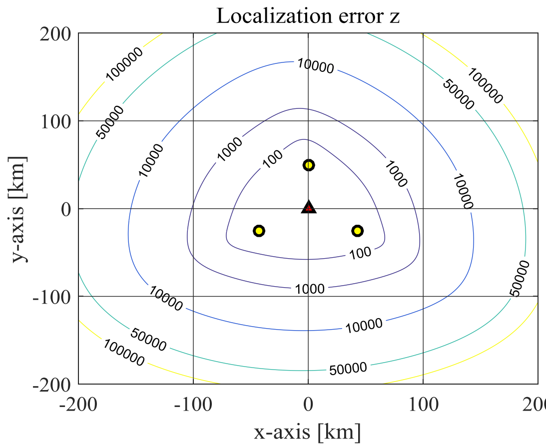

Figure 7. Localization error of *z*-axis according to target location.

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