Root-MUSIC based Multiple Orthogonal Subarrays in a Rectangular Array

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***Abstract*—This paper proposes root-MUSIC (MUltiple SIgnal Classification) based multiple orthogonal subarrays to improve the performance of the root-MUSIC algorithm in a rectangular array. Estimation of the direction of arrival (DOA) is required to estimate the incident angle of signals, and DOA estimation technique is used in various applications such as radar, sonar, and communications. There are many estimation methods of DOA such as Bartlett, Capon, MUSIC, and root-MUSIC algorithms, etc. Compared to the spectral method (Bartlett, Capon, MUSIC), the root- MUSIC, which is represented by the parameter technique, is known to have better resolution. However, since the root-MUSIC algorithm is only available in the uniform linear array (ULA), it can be used only in two ULAs such as L-arrays in two-dimensional arrays. In this paper, we propose a method of the root-MUSIC based multiple orthogonal subarrays in order to improve the performance of the root-MUSIC in a rectangular array and compares the performance with the L-shaped array.**

***Keywords*—DOA estimation; 2D-MUSIC; root-MUSIC; array signal processing;**

I. Introduction

Estimation of the direction of arrival (DOA) is required to estimate the incident angle of signals and is used in various applications such as radar, sonar, and communications [1-4]. DOA Estimation algorithms can be classified into a spectrum-based algorithm and a parameter-based algorithm. Representative spectrum-based algorithms are Bartlett and Capon algorithms based on beamforming [1-2] and MUSIC (MUltiple SIgnal Classification) algorithm using subspaces of signals [3]. The MUSIC algorithm has better DOA estimation performance than Capon and Bartlett, but still has lower resolution for two adjacent signals and long estimation time. Compared to the spectral method, the root- MUSIC, which is represented by the parameter technique, is known to have better resolution. The root-MUSIC algorithm estimates the DOA by using the root of the output spectrum of MUSIC, and has a short estimation time due to low calculation amount [4].

The Bartlett, Capon, and MUSIC algorithms can be applied to circular arrays and rectangular arrays when extending into a two-dimensional array [5-7]. However, since the root-MUSIC algorithm is only available in the ULA, it can be used only in two ULAs that are orthogonal, such as L-arrays in two-dimensional arrays [8-9]. If a 4 by 4 rectangular array is used,

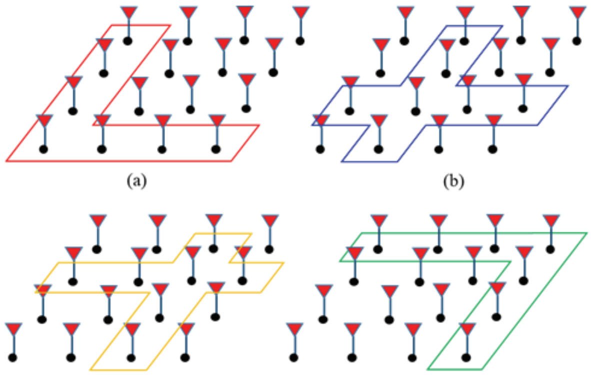

Fig. 1. Orthogonal subarray in a rectangular array

unlike the spectrum-based algorithm which uses all 16 antennas, the root-MUSIC algorithm using an L-shaped array can use only 7 antennas. It is difficult to use root-MUSIC in a rectangular array because the performance decreases when the number of antennas decreases [10]. In this paper, we propose a method of the root-MUSIC based multiple orthogonal subarrays in order to improve the performance of the root-MUSIC in a rectangular array. For example, as shown in Figure 1, four root-MUSIC results use four pairs of orthogonal subarrays in a 4 by 4 array.

II. Root-MUSIC in an L-shaped Array

A. *Signal Model* [5-7]

A number of receivers in a rectangular array are $N_x \cdot N_y$ antennas, where N_x and N_y are the number of antennas on the x-axis and y-axis, respectively. The signal received by each antenna is represented by a vector $\mathbf{r} = [\mathbf{r}_{0,0}, \mathbf{r}_{0,1}, \cdots, \mathbf{r}_{0,N_y-1}, \mathbf{r}_{1,0}, \cdots, \mathbf{r}_{1,N_y-1}, \cdots, \mathbf{r}_{N_x-1,N_y-1}]^T$ as follows:

$$\mathbf{r}(t) = \sum_{i=1}^{D} \mathbf{a}(\theta_i, \phi_i) s_i(t) + \mathbf{w}(t), \qquad (1)$$

where, $\mathbf{r}_{l,m}$ is the received signal of the antenna at position (l,m), D is the number of signals, s_i and $\mathbf{a}(\theta_i, \phi_i)$ are the signal and the steering vector of the i-th source signal, and $\mathbf{w}$ is a noise vector. θ_i and ϕ_i are azimuth and elevation angles of the i-th source signal with $\theta_i \in [0,2\pi)$ and $\phi_i \in [0,\pi/2)$. The steering vector is expressed as follows:

$$\mathbf{a}(\theta,\phi) = \left[\mathbf{a}_0(\theta,\phi)^T, \mathbf{a}_1(\theta,\phi)^T, \cdots, \mathbf{a}_{N_y-1}(\theta,\phi)^T\right]^T, \quad (2)$$

$$\mathbf{a}_m(\theta,\phi) = e^{j2\pi m\psi_y} \cdot \left[1, e^{j2\pi\psi_x}, \cdots, e^{j2\pi(N_x-1)\psi_x}\right]^T, \quad (3)$$

where $\psi_x = (d_x/\lambda)\sin\phi\cos\theta$, $\psi_y = (d_y/\lambda)\sin\phi\sin\theta$, d_x and d_y are the distances between the antennas on the x and y axes, and λ is the wavelength.

B. *MUSIC algorithm in two-dimensions* [7]

The MUSIC algorithm uses the noise subspace and the signal subspace which are orthogonal to each other. Using white Gaussian noise matrix **w**, the noise covariance is givey by $\mathbf{R}_\mathrm{w} = \sigma_w^2 \mathbf{I}_{N_x N_y}$, where, $\mathbf{I}_p$ is a p by p identity matrix, σ_w^2 is noise power. The covariance of the received signal is used to obtain the signal subspace and the noise subspace. The covariance matrix of the received signal denoted by $\mathbf{R}_\mathrm{r} = \mathbf{A}\mathbf{R}_\mathrm{s}\mathbf{A}+ \mathbf{R}_\mathrm{w}$, where $\mathbf{A} = [\mathbf{a}(\theta_1,\phi_1), \mathbf{a}(\theta_2,\phi_2), \cdots, \mathbf{a}(\theta_D,\phi_D)]$ and $\mathbf{R}_\mathrm{s}$ is the covariance matrix of the source signal $\mathbf{s} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_D]$. The eigenvector **U** can be obtained by eigenvalue decomposition of **Rr**, and the obtained eigenvector is divided into D signal subspaces $\mathbf{U}_\mathrm{s} = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_D]$ and noise subspaces $\mathbf{U}_\mathrm{w} = [\mathbf{u}_{D+1} \cdots, \mathbf{u}_{N_x N_y}]$. Since the signal subspace is orthogonal to the noise subspace and the steering vector is in the signal subspace, the MUSIC spectrum in the two-dimensions is defined as follows:

$$P_{2D,MUSIC}(\theta,\phi) = \frac{1}{\left|\mathbf{a}^H(\theta,\phi)\mathbf{U}_\mathrm{w}\right|^2}. \quad (4)$$

C. *Root-MUSIC in an L-shaped array* [8]

To use the root-MUSIC algorithm in two-dimensions, it can be used only in L-shaped arrays where two ULAs intersect orthogonally. The Root-MUSIC algorithm in the L-shaped is obtained by performing the Root-MUSIC algorithm in each ULA. For the Root-MUSIC calculation in the L-shaped array, one-dimensional MUSIC algorithm results are firstly used. The one-dimensional MUSIC algorithm is as follows:

$$P_{1D,MUSIC}(\theta_a) = \frac{1}{\left|\mathbf{a}_{1D}^H(\theta_a)\mathbf{U}_{\mathbf{w},1D}\right|^2}, \quad (5)$$

where $\mathbf{U}_{\mathbf{w},1D}$ is noise subspaces of one-dimensional signal model, $\mathbf{a}_{1D}(\theta_a) = [1, e^{j2\pi f(\theta_a)}, \cdots, e^{j2\pi(N-1)f(\theta_a)}]$ is a steering vector of one-dimensional signal model, $f(\theta_a) = (d_x/\lambda)\sin\theta_a$, θ_a is an angle of incidence of a source signal with $\theta_a \in [0,\pi)$, and N is the number of antennas in the ULA. In order to find the root of the one-dimensional MUSIC, the inverse of (5) can be expressed as

$$\begin{aligned} P_{1D,MUSIC}^{-1}(\theta_a) &= \textstyle\sum_{l=0}^{N-1}\sum_{k=0}^{N-1} e^{-j2\pi l f(\theta_a)} C_{(l+1)(k+1)} e^{j2\pi k f(\theta_a)} \\ &= \textstyle\sum_{m=-N+1}^{N-1} c_m e^{-j2\pi m f(\theta_a)}, \quad (6) \end{aligned}$$

where C_{lk} is the (l, k) element of the matrix $\mathbf{C} = \mathbf{U}_{\mathbf{w},1D}\mathbf{U}_{\mathbf{w},1D}^H$, $c_m = \sum_{l-k=m} C_{lk}$. Equation (6) is redefined using z-transform as follows:

$$D(z) = \textstyle\sum_{m=-N+1}^{N-1} c_m z^{-m}. \quad (7)$$

Since the peak of the spectrum is the same as the polynomial root on the unit circle, the root closest to the unit circle within the unit circle is determined as the root corresponding to θ_a of the source signal. θ_a from the root of the polynomial $z_r = z_r e^{j\cdot\arg(z_r)}$ is as follows:

$$\theta_a = \sin^{-1}\left[\frac{\lambda}{2\pi d_x}\arg(z_r)\right]. \quad (8)$$

The root-MUSIC of the L-shaped array can be derived by extending the one-dimensional root-music described above. The phases of the results of the root-MUSIC of each axis are ψ_x and ψ_y. For convenience of calculation, denote α and β as follows

$$\begin{aligned} \alpha &= \psi_x\left(\frac{\lambda}{d_x}\right) = \sin\phi\cos\theta, \\ \beta &= \psi_y\left(\frac{\lambda}{d_y}\right) = \sin\phi\sin\theta. \quad (9) \end{aligned}$$

θ and ϕ angle are computed through (9) as follows:

$$\theta = \sin^{-1}\left|\sqrt{\alpha^2+\beta^2}\right|, \phi = \tan^{-1}(\beta/\alpha). \quad (10)$$

III. ROOT-MUSIC WITH MULTIPLE ORTHOGONAL SUBARRAY

Most of the studies on root-MUSIC in two dimensions were performed on L-shaped arrays [8-9]. However, the root-MUSIC can be used in orthogonal subarrays as shown in Figure 1. If the root-MUSIC algorithm is used in Figure 1 (b) or (c), the steering vector is generated after setting the intersection antenna as the origin. When the second antenna is set as the origin in the ULA, the steering vector is $\mathbf{a}_{1D}(\theta_a) = [\,e^{-j2\pi f(\theta_a)}, 1, e^{j2\pi f(\theta_a)}, \cdots, e^{j2\pi(N-2)f(\theta_a)}]$. Equation (6) using the new steering vector is as follows:

$$\begin{aligned} P_{1D,MUSIC}^{-1}(\theta_a) &= \textstyle\sum_{l=-1}^{N-2}\sum_{k=-1}^{N-2} e^{-j2\pi l f(\theta_a)} C_{(l+2)(k+2)} e^{j2\pi k f(\theta_a)} \\ &= \textstyle\sum_{m=-N+1}^{N-1} c_m e^{-j2\pi m f(\theta_a)}. \quad (11) \end{aligned}$$

As the steering vector changes, the range of sigma in the first row of (11) changes but the sigma of the second row does not change. Since the next process is the same, the result of the root-MUSIC of the ULA with the first antenna as the origin is the same. Based on this, it is possible to derive the result of two-dimensional root-MUSIC using orthogonal subarray. The root-MUSIC algorithm using multiple orthogonal subarrays in a rectangular array derives the results by averaging the results of multiple azimuth and elevation using a number of orthogonal subarrays. When N orthogonal subarrays are used, N

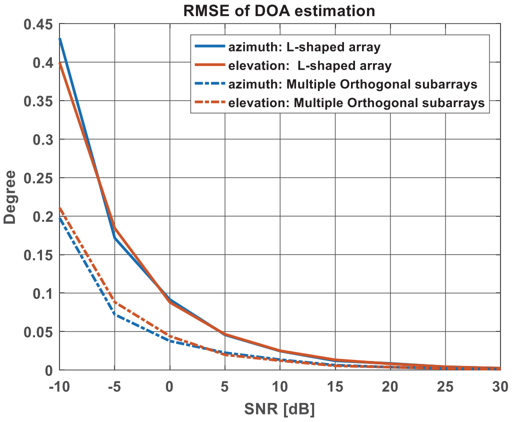

Fig. 2. RMSE performance of L-shaped array and multiple orthogonal subarrays for one incident signal - (θ_1, ϕ_1) : (40, 30)

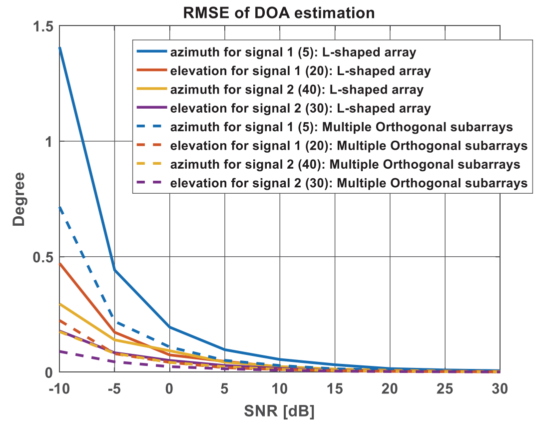

Fig. 3. RMSE performance of L-shaped array and multiple orthogonal subarrays for two incident signal - (θ_1, ϕ_1) : (40, 30), (θ_2, ϕ_2) : (5, 20).

azimuth $[\theta_{i,1}, \theta_{i,2}, \dots, \theta_{i,N}]$ and elevation $[\phi_{i,1}, \phi_{i,2}, \dots, \phi_{i,N}]$ are estimated, and finally estimated azimuth and elevation angles are as follows:

$$\theta_i = \frac{\theta_{i,1} + \cdots + \theta_{i,N}}{N}, \phi_i = \frac{\phi_{i,1} + \cdots + \phi_{i,N}}{N}. \tag{12}$$

IV. Simulation Result

We compare the performance of the L-shaped array and the multiple orthogonal subarrays in a 4 by 4 rectangular array. (d_x/λ) and (d_y/λ) are set to 0.5, and four orthogonal subarrays are used as shown in Figure 1.

Figure 2 shows the RMSE (root-mean square error) performance of L-shaped arrays and multiple orthogonal subarrays for one incident signal. The azimuth and elevation of incident signal are (40, 30) degrees. The estimation performance of multiple orthogonal subarrays is about twice as good as that of the L-shaped array.

Figure 3 shows the RMSE (root-mean square error) performance of L-shaped array and multiple orthogonal subarrays for two incident signals. The azimuth and elevation of incident signals are (40, 30), (5, 20) degrees. Similar to the results in Figure 2, the estimation performance of a multi-orthogonal sub-array is about twice as good as that of an L-type array.

V. Conclusion

We propose a method of using multiple orthogonal sub-arrays to improve Root-MUSIC performance in a rectangular array. It is shown that root-MUSIC can be used not only for L-shaped but also orthogonal subarray in a rectangular array. The use of the L-shaped array in the root-MUSIC algorithm in a rectangular array results in unused antennas, but all antennas can be used if multiple orthogonal subarrays are used. The estimation performance of a multi-orthogonal sub-array was about twice as good as that of an L-type array.

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