# Direction of Arrival Estimation in Planar Time Modulated Arrays

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***Abstract*—Time modulated arrays (TMAs) have drawn much interest for estimating direction of arrivals (DOAs) because of the simple structure. However, study on TMA has been done only in linear arrays. To extend the application of TMA, we consider it in planar array. A switching pattern of planar TMA is firstly suggested for beamforming. Then, considering its characteristics of beamforming of TMA, we estimate DOAs of the far-field sources by using multiple signal classification (MUSIC) algorithm. Simulation results show that the DOA estimation in planar TMA has the similar performance to that with RF switches but has distortion in the spatial spectrum compared to the conventional antenna planar arrays.**

**Keywords—time modulated array (TMA); direction of arrival (DOA); Planar antenna arrays;**

## I. INTRODUCTION

Time modulated array (TMA) was first proposed by Shanks and Bickmore in 1950s [1]. A TMA beam scanner can be easily implemented without the use of phase shifter, just by periodically switching on and off each array element with high-speed RF (Radio Frequency) switches. However, there is a practical limit of TMA that its beam directions are not adjustable but are pre-determined by the number of antennas. Despite the limit of TMA, there is no difficulty in estimating DOAs of the far-field sources. An approach for estimating the DOAs in a time modulated linear array (TMLA) by using the MUSIC (Multiple Signal Classification) algorithm was proposed in 2010 [2], but study on TMA for 2D array has hardly been progressed. In this paper, we extend the linear TMA to the planar array and propose an approach for DOA estimation in the planar TMA. The detail feature of array factor in the TMA is discussed in Section II. Approach of DOA estimation in planar TMA is presented in Section III.

## II. ARRAY FACTOR OF TMLA

The weight of each element in the TMLA with uniform inter-element spacing is a periodic time function. To illustrate the weight, an example of 4-element TMLA in shown in Figure 1. The weight function for $k$th element of $L$ element TMA is defined by (1)

$$w_k(t) = \begin{cases} 1, & 0 \le \tau_{k,on} < t < \tau_{k,off} \le T_p \\ 0, & \text{elsewhere,} \end{cases} \quad (1)$$

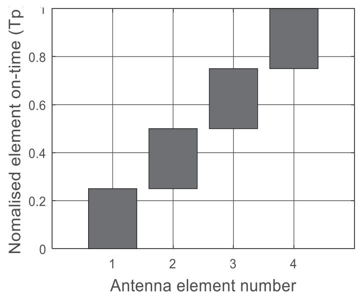


Fig. 1. Switching sequence of 4-element TMLA.

where $\tau_{k,on}$ and $\tau_{k,off}$ are on and off time of the $k$th element, $T_p$ is the switching period. Then the array factor of a $L$-element linear TMA is given by (2) [3]

$$AF(\theta,t) = e^{j2\pi f_0 t} \sum_{k=1}^{L} w_k(t) e^{j(k-1)\frac{2\pi f_0}{c} d \sin\theta}, \quad (2)$$

where $f_0$ is the center frequency, $d$ is the element spacing of the array, $c$ is the velocity of light in free space, $\theta$ is an angle with respect to the broadside direction. Since a periodic time function can be represented by the Fourier series, (2) can be expressed as (3)

$$AF(\theta,t) = \sum_{p=-\infty}^{\infty} e^{j2\pi(f_0+f_p)t} \cdot \sum_{k=1}^{L} b_{p,k} e^{j(k-1)\frac{2\pi f_0}{c} d \sin\theta}, \quad (3)$$

where $b_{p,k}$ represents the Fourier series coefficient of $w_k(t)$, and is given by (4)

$$b_{p,k} = \left(\tau_{k,off} - \tau_{k,on}\right) \frac{\sin(\pi p(\tau_{k,off}-\tau_{k,on}))}{mp(\tau_{k,off}-\tau_{k,on})} e^{-j\pi p(\tau_{k,off}+\tau_{k,on})}. \quad (4)$$

The Fourier coefficients can be considered as complex weights, the term $e^{-j\pi p(\tau_{k,off}+\tau_{k,on})}$ regard as a phase shifter at the harmonic frequencies defined by $p$.

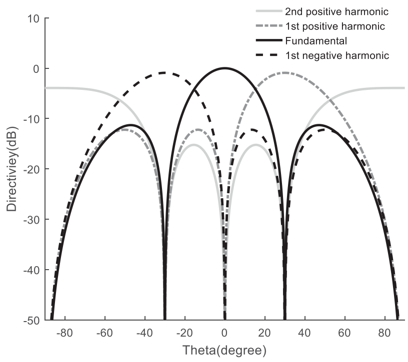


Fig. 2. Array factor produced by the switching pattern in Figure 1 (2nd negative harmonic array factor is same with 2nd positive harmonic array factor).

From (3), directions at each harmonic frequency are determined by $\theta = \sin^{-1}(2p/L)$ with $d = 0.5\lambda$, where $\lambda$ is a wavelength. This implies that the number and values of directions of the array factor in TMLA are determined only by the number of element $L$. An example of 4-element TMLA with $d = 0.5\lambda$, radiation patterns at different harmonic frequencies are shown in Figure 2. Based on the analysis above, the approach for the DOA estimation in the planar TMA will be presented in Section III.

## III. DOA ESTIMATION IN PLANAR TMA

In $N\times M$ planar TMA, all elements are equally spaced and for example, a scheme and corresponding time sequence of $4\times4$ planar TMA are shown in Figure 3 and Figure 4. Element number $k$ is defined as $(M\cdot m+n)$, corresponding to element location number $n = 0,1,..,N$ at $x$-axis and $m = 0,1,..,M$ at $y$-axis. Then, suppose that there are $D$ far-field narrowband sources with the same carrier frequency $f_0$ and sources are noncoherent.

The received signal on the $N\times M$ planar TMA is given by (5)

$$x(t) = \sum_{k=1}^{NM} w_k(t)\cdot \left[\sum_i^D s_i(t)\cdot e^{j(n\psi_x(\theta_i,\psi_i)+m\psi_y(\theta_i,\psi_i))} + n_k(t)\right], \quad (5)$$

where $s_i(t)$ is the signal emitted by the $i$th far-field source, and $\psi_x(\theta,\psi)$, $\psi_y(\theta,\psi)$ are given by (6),(7)

$$\psi_x(\theta,\psi) = \frac{2\pi}{\lambda} d_x \sin\theta\cos\phi, \quad (6)$$

$$\psi_y(\theta,\psi) = \frac{2\pi}{\lambda} d_y \sin\theta\sin\phi, \quad (7)$$

where $d_x$, $d_y$ are the $x$-axis and $y$-axis element spacing of the array, $\phi$ is an azimuth and $\theta$ is an elevation with respect to the broadside direction, $n_k(t)$ denotes the zero-mean white Gaussian noise with variance of $\sigma^2$.

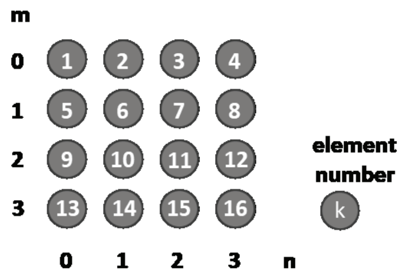


Fig. 3. Scheme of $\boldsymbol{N}\times\boldsymbol{M}$ planar TMA.

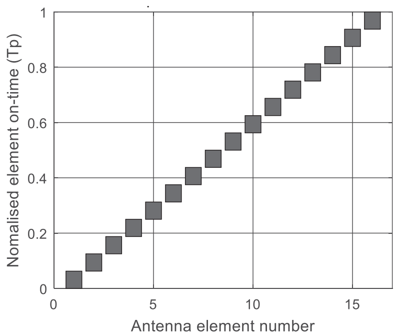


Fig. 4. Switching sequence of $\mathbf{4}\times\mathbf{4}$ planar TMA (Figure 3).

Based on the previous analysis, (5) can be rewritten as (8)

$$x(t) = \sum_{i=1}^{D}\sum_{k=1}^{NM} b_{p,k}\cdot \left[s_i(t)e^{j2\pi p f_p t}e^{j\left((n\psi_x(\theta_i,\psi_i)+m\psi_y(\theta_i,\psi_i)\right)} + n_k(t)\right]. \quad (8)$$

From (8) the received signal is sum of fundamental frequency component and harmonic frequency components. The received signal can be separated by band-pass filters, and converted into the same IF (Intermediate Frequency) stage. then the output signal obtained as (9) [2]

$$y(t) = \sum_{i=1}^{D}\sum_{k=1}^{NM} b_{q,k}\cdot \left[s_i^{IF}(t)\cdot e^{j((n\psi_x(\theta_i,\psi_i)+m\psi_y(\theta_i,\psi_i))} + n_k(t)\right], \quad (9)$$

where $s_i^{IF}(t)$ denotes the IF signal of $s_i(t)$. Equation (9) can be rewritten as a matrix form, given by (10)

$$\mathbf{Y}[n] = \mathbf{B}^T(\mathbf{A}(\boldsymbol{\psi})\mathbf{S}[n] + \mathbf{N}[n]), \quad (\mathrm{n} = 1,2,\dots,N_s), \quad (10)$$

where $[\cdot]^T$ denotes the transpose, $N_s$ is the number of samples and,

$$\mathbf{Y}[n] = \left[y_{-Q}[n], y_{-Q+1}[n], \dots, y_Q[n]\right]^T \quad (11)$$

$$\mathbf{S}[n] = \left[s_1^{IF}[n], s_2^{IF}[n], \dots, s_D^{IF}[n]\right]^T \quad (12)$$

$$\mathbf{N}[n] =$$

$$\left[n_{0,0}[n], \dots, n_{0,N-1}[n], n_{1,0}[n], \dots, n_{M-1,N-1}[n]\right]^T, \quad (13)$$

$$\mathbf{A}(\boldsymbol{\psi}) = [\mathbf{a}(\boldsymbol{\psi}_1), .., \mathbf{a}(\boldsymbol{\psi}_D)], \quad (14)$$

$$\mathbf{a}(\boldsymbol{\psi}) = vec[\boldsymbol{A}_{mat}(\boldsymbol{\psi})] =$$

$$[\mathbf{a}_0(\boldsymbol{\psi}), \dots, \mathbf{a}_{M-1}(\boldsymbol{\psi})]^T \quad (15)$$

$$\mathbf{A}_{mat}(\boldsymbol{\psi}) = [\mathbf{a}_0(\boldsymbol{\psi}), \dots, \mathbf{a}_{M-1}(\boldsymbol{\psi})], \quad (16)$$

$$\mathbf{a}_m(\boldsymbol{\psi}) = \left[e^{jm\psi_y}, e^{j(\psi_x + m\psi_y)}, \dots, e^{j((N-1)\psi_x + m\psi_y)}\right]^T, \quad (17)$$

$$\boldsymbol{\psi} = \begin{bmatrix} \psi_x \\ \psi_y \end{bmatrix}, \quad (18)$$

and the matrix **B** is Fourier coefficient matrix given by (19)

$$\mathbf{B} = \begin{bmatrix} b_{-Q,1} & b_{-Q+1,1} & \cdots & b_{Q,1} \\ b_{-Q,2} & b_{-Q+1,2} & \cdots & b_{Q,2} \\ \vdots & \vdots & \ddots & \vdots \\ b_{-Q,NM} & b_{-Q+1,NM} & \cdots & b_{Q,NM} \end{bmatrix}, \quad (19)$$

where $Q$ is positive integer that represent the maximum order of harmonic $q$.

Then we can apply the MUSIC algorithm to the covariance matrix **Y** [4], the spatial spectrum is given by (20)

$$P(\boldsymbol{\psi}) = \frac{1}{\mathbf{A}(\boldsymbol{\psi})^H (\mathbf{B}^T)^H \mathbf{E}_N \mathbf{E}_N^H \mathbf{B}^T \mathbf{A}(\boldsymbol{\psi})} \quad (20)$$

where $[\cdot]^H$ denotes the complex conjugate transpose and $\mathbf{E}_N$ is the noise subspace.

## IV. Simulation Results

Consider a 4×4 planar TMA of isotropic elements with half a wavelength uniform spacing like Figure 3. Suppose that there are three non-coherent far-field narrowband signals with same frequency, equal power, arriving from $(\phi_{x1}, \phi_{y1}) = (-45°, 26°)$, $(\phi_{x2}, \phi_{y2}) = (26°, -15°)$, $(\phi_{x3}, \phi_{y3}) = (-10°, 37°)$, where $(\phi_x, \phi_y)$ represent a horizontal angle and a vertical angle for convenient observation. A result of Spatial spectrum of the TMA is shown in Figure 5 and the result of the conventional MUSIC in Figure 6. The Spatial spectrum of the TMA has distortion compared to the conventional MUSIC, but it shows that the distortion has little effect to an estimation performance.

## V. Conclusion

We extended a linear TMA to a planar one and showed that DOAs can be estimated in the planar TMA by using the MUSIC algorithm. In spatial spectrum of planar TMA MUSIC, however, there is some distortion compared to the conventional MUSIC. This distortion will be analyzed in the future works. Furthermore, study on new switching patterns and array shapes of TMA is also needed for better DOA estimation performance.

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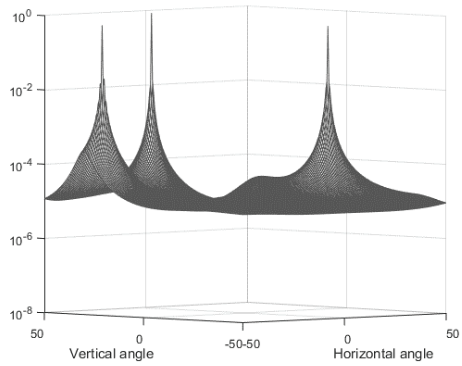


Fig. 5. Spatial spectrum of the 4×4 planar TMA (Figure 3) MUSIC.

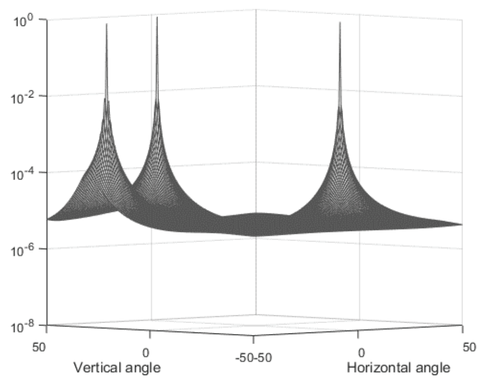


Fig. 6. Spatial spectrum of the conventional planar MUSIC.