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PERFORMANCE ANALYSIS ON A DECOUPLED MAXIMUM LIKELIHOOD ANGLE ESTIMATOR IN AN FM BASED PASSIVE BISTATIC RADAR

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ABSTRACT
We analyze the performance of a decoupled maximum likelihood (DEML) angle estimator in an FM based passive bistatic radar. Under the assumption that the bistatic range, the Doppler frequency estimates, and the transmitted signal emitted from an illuminator are given, we analyze the sensitivity of the DEML estimator against the range and Doppler frequency errors from the analytic expressions and simulation results. As a result, the performance of the DEML estimator may be mainly degraded by the Doppler frequency errors and we derive a condition that the root-mean-square error of the DEML estimator diverges.

KEY WORDS
Decoupled maximum likelihood angle estimator, FM based bistatic radar.

1. Introduction

A passive bistatic radar exploits the non-radar transmitters of opportunity, such as FM (frequency modulated) radio, DVB (digital video broadcasting), DAB (digital audio broadcasting), and GSM (global system for mobile communications) for detecting the multiple fast moving targets [1-2, 8-11]. In order to detect K targets, the relative bistatic range, Doppler frequency, and angle-of-arrival estimates corresponding to each target should be extracted from the received signals. The angle-of-arrival estimates can be extracted from an antenna array with M sensors. The maximum likelihood approach may be considered to estimate the parameters [3]. Since the maximum likelihood approach performs a K-dimensional search [3], however, the computational burden in this search problem is dramatically increased according to the number of targets.

To cope with this problem, a decoupled maximum likelihood (DEML) angle estimator [4] was proposed to improve the computational complexity of the maximum likelihood technique in [3] by using an assumption that a *priori* information of the multiple source signals are given; that is, the source signals are assumed to be known. Thus, the DEML estimator can be easily applied to the communication systems by using the preamble information. After that, several joint estimation techniques [5-7] which can estimate the range, the Doppler shift, and the angle-of-arrival, simultaneously, were suggested. However, since the range and the Doppler frequency values are given in some applications, these joint estimation algorithms may not be effective for such applications as the passive radar systems because of the computational burden of these joint estimation techniques. Since the DEML angle estimator estimates only angle values, we have focused on the DEML estimator to utilize the simplicity of the DEML angle estimator.

When we consider applying the DEML angle estimator in passive radar areas, however, it is difficult to use a *priori* information of incident signals directly. Since the target echo signals are time-delayed and Doppler frequency shifted versions of the transmitted signal, the source signals should be obtained from the estimates of the relative bistatic range and Doppler frequency. In case of using the estimates of the source signals, the relative bistatic range and Doppler frequency errors may cause a critical problem in the DEML angle estimator.

In this paper, we investigate the feasibility of the DEML angle estimator when we have the range and Doppler frequency estimates with the critical errors. To figure out the sensitivity of the DEML estimator against the range and Doppler frequency errors, we analyze the effect of the range and Doppler frequency errors from several analytic expressions and simulation results.

This paper is organized as follows. Section 2 presents the signal models of a received signals and defines the problem of interest. In section 3, the DEML angle estimator is briefly reviewed, and the performance analysis of the DEML angle estimator is covered in section 4. Simulation results are presented in section 5 and conclusions are drawn in section 6.

2. Problem Formulation

Consider the angle estimation of K narrowband target echoes $s_k(t)$ ($k = 1,\dots, K$) impinging on an arbitrary array with M sensors. Under the assumption that the interferences are eliminated, then a received signal $\mathbf{x}(t)$ is given by

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k, \phi_k) s_k(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{a}(\theta_k, \phi_k)$ denotes the steering vector corresponding to the elevation θ_k and azimuth angle ϕ_k of the kth target echo signal, and $\mathbf{n}(t)$ is a noise vector of the white

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Gaussian random process with covariance matrix $\mathbf{Q}=\sigma_n^2\mathbf{I}_M$. The above equation can be rewritten as

$$\mathbf{x}(t)=\sum_{k=1}^{K}\mathbf{a}(\theta_k,\phi_k)\gamma_k y(t-\tau_k)e^{-j2\pi f_k t}+\mathbf{n}(t), \qquad (2)$$

where $y(t)$ denotes the complex envelope of the transmitted FM signal, γ_k is the amplitude of the kth target echo. τ_k is the relative bistatic range, and f_k represents the relative Doppler frequency of the kth target echo. The received signal vector $\mathbf{x}(t)$ can be reduced to a matrix form as follows:

$$\begin{aligned}\mathbf{x}(t)&=\mathbf{A}\boldsymbol{\Gamma}\mathbf{y}(t)+\mathbf{n}(t)\\&=\mathbf{B}\mathbf{y}(t)+\mathbf{n}(t),\end{aligned} \qquad (3)$$

where $\mathbf{A}=\left[\mathbf{a}(\theta_1,\phi_1),\cdots,\mathbf{a}(\theta_K,\phi_K)\right]$ denotes the array manifold matrix, $\boldsymbol{\Gamma}=\mathrm{diag}\{\gamma_1,\cdots,\gamma_K\}$ represents an amplitude matrix, and the kth entry of the $\mathbf{y}(t)$ is defined as $y(t-\tau_k)e^{-j2\pi f_k t}$.

Given a *priori* information of $y(t)$, the problem of interest is to estimate the parameters $\{\tau_k, f_k, \theta_k, \phi_k\}_{k=1,\dots,K}$ for the target tracking in the passive bistatic radar systems. Since the relative bistatic range and Doppler frequency estimates may be easily estimated from the cross-correlation function of $y(t)$ and $\mathbf{x}(t)$, in this paper, the estimates of bistatic range $\hat{\tau}_k$ and bistatic Doppler frequency $\hat{f}_k$ are assumed to be given. Thus, the problem considering in this paper, can be defined to estimate $\{\theta_k,\phi_k\}_{k=1,\cdots,K}$ by using $y(t)$, $\hat{\tau}_k$ and $\hat{f}_k$ from N data samples $\{\mathbf{x}(t_n)\}_{n=1,\cdots,N}$.

3. Decoupled Maximum Likelihood (DEML) Angle Estimator [4]

The decoupled maximum likelihood (DEML) angle estimator [4] determines the direction-of-arrival $\{\theta_k,\phi_k\}_{k=1,\cdots,K}$ and the amplitude $\{\gamma_k\}_{k=1,\cdots,K}$ of each target echo signal, separately. Since the DEML estimator performs K 2-dimensional search (if the parameter of interest is only the azimuth angle, then the DEML estimator performs K 1-dimensional search), the DEML estimator reduces the computational complexity of the ML estimator which performs the K-dimensional search. Note that the reference signal vector $\mathbf{y}(t)$ is assumed to be known; that is, $y(t)$ and $\{\theta_k,\phi_k\}_{k=1,\dots,K}$ are given, in this section.

When N samples of the received signal $\{\mathbf{x}(t_n)\}_{n=0,\cdots,N-1}$ are obtained, the estimate of the noise covariance matrix $\hat{\mathbf{Q}}$ can be calculated by

$$\hat{\mathbf{Q}}=\hat{\mathbf{R}}_{xx}-\hat{\mathbf{R}}_{yx}^H\hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{R}}_{yx}, \qquad (4)$$

where

$$\hat{\mathbf{R}}_{yy}=\frac{1}{N}\sum_{n=0}^{N-1}\mathbf{x}(t_n)\mathbf{x}^H(t_n), \qquad (5)$$

and

$$\hat{\mathbf{R}}_{yx}=\frac{1}{N}\sum_{n=0}^{N-1}\mathbf{y}(t_n)\mathbf{x}^H(t_n). \qquad (6)$$

$\hat{\mathbf{R}}_{xx}$ may be easily obtained similarly as $\hat{\mathbf{R}}_{yy}$. By using (5) and (6), the estimate of the product of $\mathbf{A}$ and $\boldsymbol{\Gamma}$; that is, $\hat{\mathbf{B}}$, may be obtained by

$$\hat{\mathbf{B}}=\hat{\mathbf{R}}_{yx}^H\hat{\mathbf{R}}_{yy}^{-1}. \qquad (7)$$

Finally, the DEML angle estimates $\hat{\boldsymbol{\Theta}}_k=\{\hat{\theta}_k,\hat{\phi}_k\}$ can be determined by

$$\begin{aligned}\hat{\boldsymbol{\Theta}}&=\arg\min_{\boldsymbol{\Theta}}\frac{\left|\mathbf{a}^H(\boldsymbol{\Theta})\hat{\mathbf{Q}}^{-1}\hat{\mathbf{b}}_k\right|}{\mathbf{a}^H(\boldsymbol{\Theta})\hat{\mathbf{Q}}^{-1}\mathbf{a}_k(\boldsymbol{\Theta})}\\&=\arg\max_{\boldsymbol{\Theta}}\ \hat{\mathbf{b}}_k^H\hat{\mathbf{Q}}^{-1/2}\mathbf{P}_{\breve{\mathbf{a}}_k}^{\perp}\hat{\mathbf{Q}}^{-1/2}\hat{\mathbf{b}}_k,\end{aligned} \qquad (8)$$

where $\hat{\mathbf{b}}_k$ denotes the kth column vector of $\hat{\mathbf{B}}$,

$$\breve{\mathbf{a}}_k=\hat{\mathbf{Q}}^{-1/2}\mathbf{a}_k, \qquad (9)$$

and

$$\mathbf{P}_{\breve{\mathbf{a}}_k}^{\perp}=\mathbf{I}_M-\breve{\mathbf{a}}_k(\breve{\mathbf{a}}_k^H\breve{\mathbf{a}}_k)^{-1}\breve{\mathbf{a}}_k^H. \qquad (10)$$

$\mathbf{I}_M$ denotes the $M\times M$ identity matrix in (10). The detailed descriptions and the statistical analysis of the DEML angle estimator can be referred in [4].

4. Performance Analysis of the DEML estimator in a bistatic radar system

Suppose that K target echo signals $\mathbf{y}(t)$ are unknown, but the estimate of $y(t)$, $\hat{\tau}_k$ and $\hat{f}_k$ are given. Note that the transmitted FM signal $y(t)$ is different from $\mathbf{y}(t)$, which denotes K time-delayed and Doppler frequency shifted versions of $y(t)$. By using $y(t)$, $\hat{\tau}_k$ and $\hat{f}_k$, the estimate of a target echo signal vector $\tilde{\mathbf{y}}(t)$ can be obtained by

$$\tilde{\mathbf{y}}(t)=\left[y_{\hat{\tau}_1}(t)e^{-j2\pi\hat{f}_1 t},\cdots,y_{\hat{\tau}_K}(t)e^{-j2\pi\hat{f}_K t}\right]^T, \qquad (11)$$

where

$$y_{\hat{\tau}_k}(t)=y(t-\hat{\tau}_k). \qquad (12)$$

In this section, when $\tilde{\mathbf{y}}(t)$ is used to estimate the incident angles instead of $\mathbf{y}(t)$ in the DEML estimator, the performance of the DEML estimator is analyzed.

Consider $\hat{\tau}_k$ and $\hat{f}_k$ are given as

$$\hat{\tau}_k = \tau_k + \tilde{\tau}_k, \quad (13)$$

$$\hat{f}_k = f_k + \tilde{f}_k, \quad (14)$$

where $\tilde{\tau}_k$ and $\tilde{f}_k$ denote the estimation error of the relative bistatic range and Doppler frequency. In order to investigate the effect of $\tilde{\tau}_k$ and $\tilde{f}_k$ respectively, the estimation error of each parameter is assumed to be zero.

Let us consider $\tilde{\tau}_k$ to be zero and $\tilde{f}_k$ to be non-zero. The estimate of a target echo signal vector $\tilde{\mathbf{y}}(t)$ can be rewritten as

$$\begin{aligned}\tilde{\mathbf{y}}(t) &= \left[y(t)e^{-j2\pi \hat{f}_1 t}, \cdots, y(t)e^{-j2\pi \hat{f}_K t} \right]^T \\ &= \mathbf{\Phi}^t \mathbf{y}(t),\end{aligned} \quad (15)$$

where $\mathbf{\Phi} = \mathrm{diag}\{e^{-j2\pi f_1}, \cdots, e^{-j2\pi f_K}\}$ denotes the Doppler frequency error matrix. When a covariance matrix of $\tilde{\mathbf{y}}(t)$ and $\mathbf{x}(t)$ is denoted by $\tilde{\mathbf{R}}_{yx}$, then $\tilde{\mathbf{R}}_{yx}$ may be represented by

$$\begin{aligned}\tilde{\mathbf{R}}_{yx} &= \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}(t_n)\mathbf{x}^H(t_n) \\ &= \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{\Phi}^{t_n} \hat{\mathbf{R}}_{yx}.\end{aligned} \quad (16)$$

Under the assumption that the target echo signals are uncorrelated, then the auto-covariance matrix of $\tilde{\mathbf{y}}(t)$ may be defined by

$$\begin{aligned}\tilde{\mathbf{R}}_{yy} &= \frac{1}{N}\sum_{n=0}^{N-1} \tilde{\mathbf{y}}(t_n)\tilde{\mathbf{y}}^H(t_n) = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{\Phi}^{t_n} \hat{\mathbf{R}}_{yy} \mathbf{\Phi}^{-t_n} \\ &= \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{\Phi}^{t_n} \mathbf{\Phi}^{-t_n} \hat{\mathbf{R}}_{yy} = \hat{\mathbf{R}}_{yy}.\end{aligned} \quad (17)$$

Since $\hat{\mathbf{R}}_{yy}$ and $\mathbf{\Phi}^{t_n}$ are the diagonal matrices, $\tilde{\mathbf{R}}_{yy}$ can be reduced to $\hat{\mathbf{R}}_{yy}$. Thus, the Doppler frequency estimation error $\tilde{f}_k$ causes critical errors in $\tilde{\mathbf{R}}_{yx}$.

Suppose that the diagonal entries of the Doppler frequency error matrix $\mathbf{\Phi}$ is a non-zero valued. When the signals are sampled with sampling frequency f_s, then $\tilde{\mathbf{R}}_{yx}$ may be rewritten as

$$\begin{aligned}\tilde{\mathbf{R}}_{yx} &= \frac{1}{N}\left(\sum_{n=0}^{N-1} \mathbf{\Phi}^{t_n} \right) \hat{\mathbf{R}}_{yx} \\ &= \frac{1}{N}(\mathbf{I}_M - \mathbf{\Phi}^N)(\mathbf{I}_M - \mathbf{\Phi})^{-1} \hat{\mathbf{R}}_{yx}.\end{aligned} \quad (18)$$

Since the inverse matrix of $(\mathbf{I}_M - \mathbf{\Phi})$ always exist, the part of the diagonal components in $\tilde{\mathbf{R}}_{yx}$ may be zero-valued as

$$\exp\left(-j2\pi \frac{\tilde{f}_k}{f_s} N \right) = 1. \quad (19)$$

Thus, if the condition $N\tilde{f}_k / f_s = q \ (q = 1, 2, \cdots)$ is satisfied, then $\tilde{\mathbf{R}}_{yx}$ may become a zero-valued matrix and it causes the critical problems in the DEML angle estimator. Since the sampling frequency f_s and the observation samples N are predetermined values in a bistatic radar system, these parameters can be controlled. However, it is difficult to predict where the critical points are occurred because the Doppler frequency estimation error $\tilde{f}_k$ is unknown. Furthermore, if the minimum critical point $(q = 1)$ approaches zero in bistatic Doppler shift error domain, the root-mean-square error (RMSE) of the azimuth angle may diverge even by relatively low Doppler frequency errors.

5. Simulation Result

In this section, we present the RMSE of the azimuth angles of the target echoes according to the relative bistatic range and Doppler frequency errors. In order to compute the RMSE of the azimuth angle, the performance of the DEML angle estimator is analyzed by the simulation program using MATLAB. First, we assume that two fast moving targets, a receiver, and an FM transmitter exist. Based on the geolocation of each platform, we computed the bistatic range and Doppler frequency values. The antenna array with 8 elements was used and the sensors were uniformly placed on a circle with a radius of 1.2 meter. The signal-to-noise ratio (SNR) of the target echo signals were assumed to be -25 dB and -20 dB, respectively. The main parameters are shown in Table 1.

The RMSE of the azimuth angle estimation is depicted in Figure 1. As referred in Section 4, when (19) is satisfied, the RMSE dramatically increases as shown in Figure 1. The bistatic Doppler frequency error $\tilde{f}_k$ which

Table 1
Simulation parameters

FM transmitter location [km]	(50, 0)
Receiver location [km]	(−50, 0)
Target location [km]	(60, 110), (−10, 80)
Target velocity [m/s]	340
Carrier frequency [MHz]	93.6
Sampling frequency f_s [kHz]	250
Observation time [ms]	40
The number of samples N	10000
Signal-to-noise ratio [dB]	−25, −20
The number of antennas	8
Antenna configuration	Circular array (radius : 1.2 meter)

generates the local maximum in the RMSE graphs, can be calculated by

$$\tilde{f}_k = \frac{f_s}{N} q = 25q, \tag{20}$$

where $f_s = 250\,\text{kHz}$, $N = 10000$, and q denotes an integer number. From (20), we can notice that the RMSE dramatically increased where $\tilde{f}_k = 25q$ is satisfied. On the other hand, the RMSE graph is insensitive to the relative bistatic range errors as shown in figure 2.

6. Conclusion

In this paper, we analyzed the performance of the DEML angle estimator in bistatic radar systems exploiting the transmitters of opportunity. Based on the analytic expressions and the simulation results, we have shown that there exist the critical points where the performance of the DEML estimator critically degraded. Furthermore, we have derived a simple formula for calculating the critical points, which is related to the sampling frequency and the number of samples. On the other hand, we have shown that the relative bistatic range error has minor impact on the DEML angle estimator compared to the relative bistatic Doppler frequency error from the simulation results. Considering that the transmitted FM signals are continuous waves and the Doppler shift estimate error is relatively small compared to the pulse train signals, we can use angle estimates in passive radar systems with the DEML estimator. This may result in improving the performance of the target localization algorithms.

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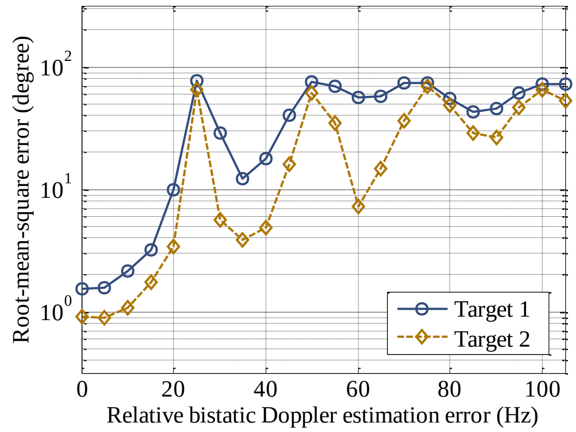

Figure 1. Root-mean-square error according to the relative bistatic Doppler frequency estimation error.

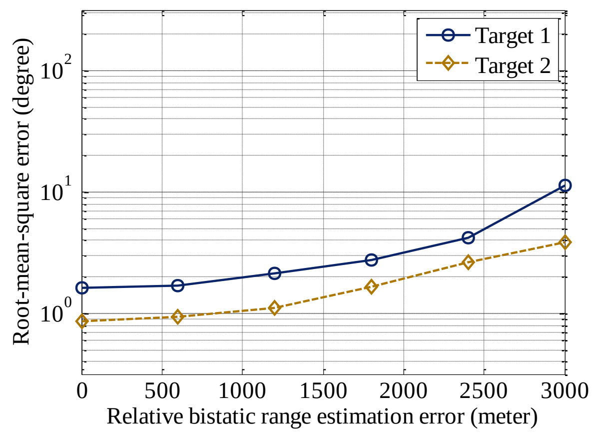

Figure 2. Root-mean-square error according to the relative bistatic range estimation error.

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