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CRLB-Based Performance Analysis of TDOA/FDOA Estimation of Communication Signals

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Abstract

A study on emitter localization using TDOA (time difference of arrival) and FDOA (frequency difference of arrival) has recently increased for electronic warfare (EW) systems. A TDOA/FDOA system generally consists of two parts: TDOA/FDOA estimation from unknown signals (extraction part); and localization from extracted TDOA/FDOA measurements (localization part). Thus, TDOA/FDOA estimation significantly affects the final performance of emitter localization. In EW systems, the information about the received signal, such as a modulation method and a symbol rate which are related to the estimation performance, is usually unknown. Consequently, we need to analyze how the estimation performance varies with a modulation method and other parameters, such as frequency deviation and a symbol rate. To achieve this, we carry out the theoretical analysis on the estimation performance according to various modulation methods and the main parameters of communication signals.

Keyword: TDOA, FDOA, Localization, CRLB, Electronic Warfare

1. Introduction

In electronic warfare systems, estimating the position of uncooperative signals is quite important for surveillance and planning military strategies.

TDOA/FDOA emitter localization methods have drawn much interest due to their higher performance in the electronic warfare system [1][2]. A TDOA/FDOA localization system generally consists of two parts: TDOA/FDOA estimation from unknown signals (extraction part), and localization using extracted TDOA/FDOA measurements (localization part) [1]. Suppose that two sensors located far from each other intercept a communication signal of an unknown emitter, the received signals at each sensor are time-shifted and frequency-shifted as follows:

$$\begin{aligned} r_1(t) &= s(t-\tau_1)e^{j2\pi\nu_1 t} + n_1(t) \\ r_2(t) &= s(t-\tau_2)e^{j2\pi\nu_2 t} + n_2(t), \end{aligned} \tag{1}$$

where $s(t)$ is a target signal emitted from an unknown emitter, and τ_1 and τ_2 are time delays from the unknown emitter to each sensor, respectively. ν_1 and ν_2 are Doppler frequencies and $n_1(t)$ and $n_2(t)$ are additive white Gaussian noise at each sensor.

Since the two sensors are located far from each other, we have to gather the received signals in one sensor or a central unit; and then acquire TDOA and FDOA information using various methods [3]-[5]. Unfortunately, the preliminary information of the unknown communication signals, such as a modulation method, a symbol rate, and amplitude, is usually unknown. However, the estimation performance may be affected by such information and thus we need to analyze how the estimation performance varies with a modulation method and other parameters in order for the algorithm to properly operate. In this sense, we carry out the theoretical analysis on the estimation performance according to a modulation method and main communication parameters by using the Cramer-Rao lower bound.

2. Performance Analysis Based on Cramer-Rao Lower Bound and Simulation

2.1 Cramer-Rao Lower Bound for TDOA and FDOA Measurements

In this subsection we explain the Cramer-Rao lower bound (CRLB) for TDOA and FDOA measurements. TDOA and FDOA are defined respectively as follows:

$$\begin{aligned} \tau_d &= \tau_1 - \tau_2 \\ \nu_d &= \nu_1 - \nu_2 \end{aligned} \tag{2}$$

Let $\hat{\tau}_d$ and $\hat{\nu}_d$ be the estimates of TDOA and FDOA. Using an estimator of $g(\cdot)$ from the received signals of $r_1(t)$ and $r_2(t)$ gives the estimates of TDOA and FDOA as follows:

$$(\hat{\tau}_d, \hat{\nu}_d) = g(r_1(t), r_2(t)) \tag{3}$$

Since the CRLB represents a lower bound on the variance of the estimate, each variance of the estimates of TDOA and FDOA is bounded by each CRLB, which was derived in [6] and [7], as follows:

$$\mathrm{var}(\hat{\tau}_d) \geq \frac{1}{4\pi^2 BT\gamma B_{rms}^2} \tag{4}$$

$$\mathrm{var}(\hat{\nu}_d) \geq \frac{1}{4\pi^2 BT\gamma T_e^2}, \tag{5}$$

where B is noise bandwidth, T is collection time, and γ is effective signal to noise ratio (SNR) defined by

$$\frac{1}{\gamma} = \frac{1}{2}\left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_1\gamma_2}\right], \tag{6}$$

where γ_1 and γ_2 are SNRs at sensor1 and sensor2, respectively. B_{rms} is root mean square (RMS) bandwidth and T_{rms} is RMS collection time, defined as follows:

$$B_{rms} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |S(f)|^2 \, df}{\int_{-\infty}^{\infty} |S(f)|^2 \, df}}$$
$$T_{rms} = \sqrt{\frac{\int_0^T t^2 |s(t)|^2 \, dt}{\int_0^T |s(t)|^2 \, dt}} \tag{7}$$

If the noise bandwidth is the same with the sampling frequency, BT becomes the number of collected samples. Then, from (4) we can find that TDOA performance depends on the signal bandwidth and the number of samples. On the other hand, from (5) we can find that FDOA performance depends on the number of samples and collection time.

2.2 Simulation Results

In this subsection, we carry out simulations to verify the theoretical performance for TDOA and FDOA estimates according to a modulation method and main communication parameters affecting the RMS bandwidth.

Figure 1 shows the CRLBs for TDOA estimates when various modulation methods, such as AM, FM, FSK, PSK and QAM are used. Note that FM, FSK, and PSK and QAM were applied in various ways by changing frequency deviation, frequency separation, and symbol rate, respectively. The collection time was set to be T=5ms. Under the same environments,

Figure 2 shows the CRLBs for FDOA estimates. The frequency deviation for FM, the frequency separation for FSK, and the symbol rate for PSK and QAM become a dominant factor to affect the performance of estimating TDOA because they determine the signal bandwidth. The results shown in Figure 1 confirm the relationship between the performance and the bandwidth. That is, the estimation performance for TDOA is improved as the frequency deviation of FM, the frequency separation of FSK, and the symbol rate of

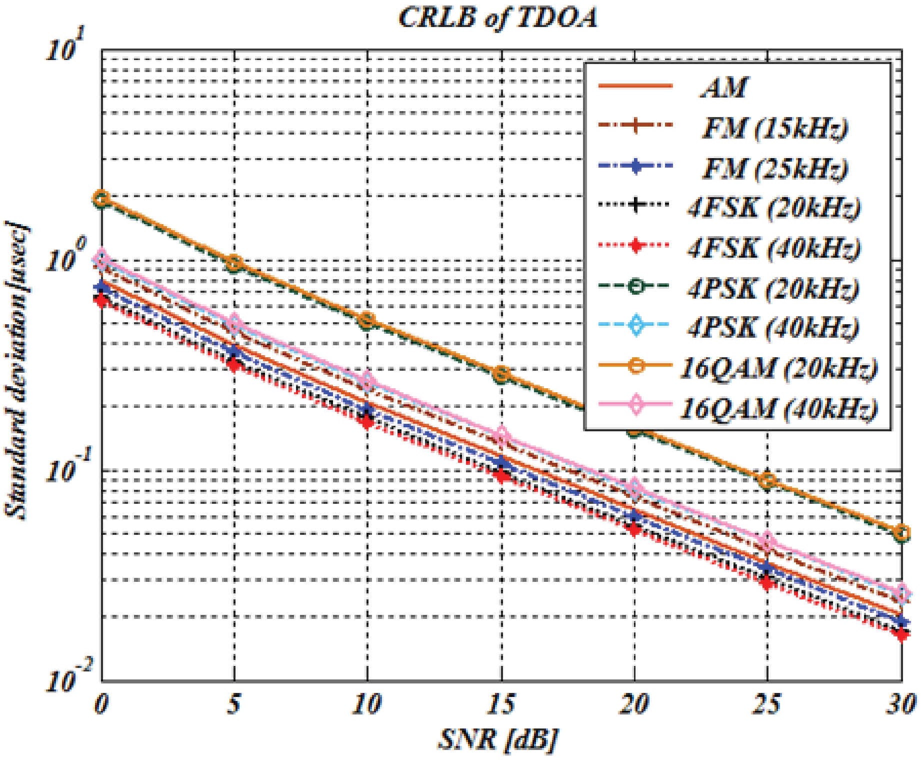

Fig. 1: Cramer-Rao lower bound of TDOA as modulation method and several parameters.

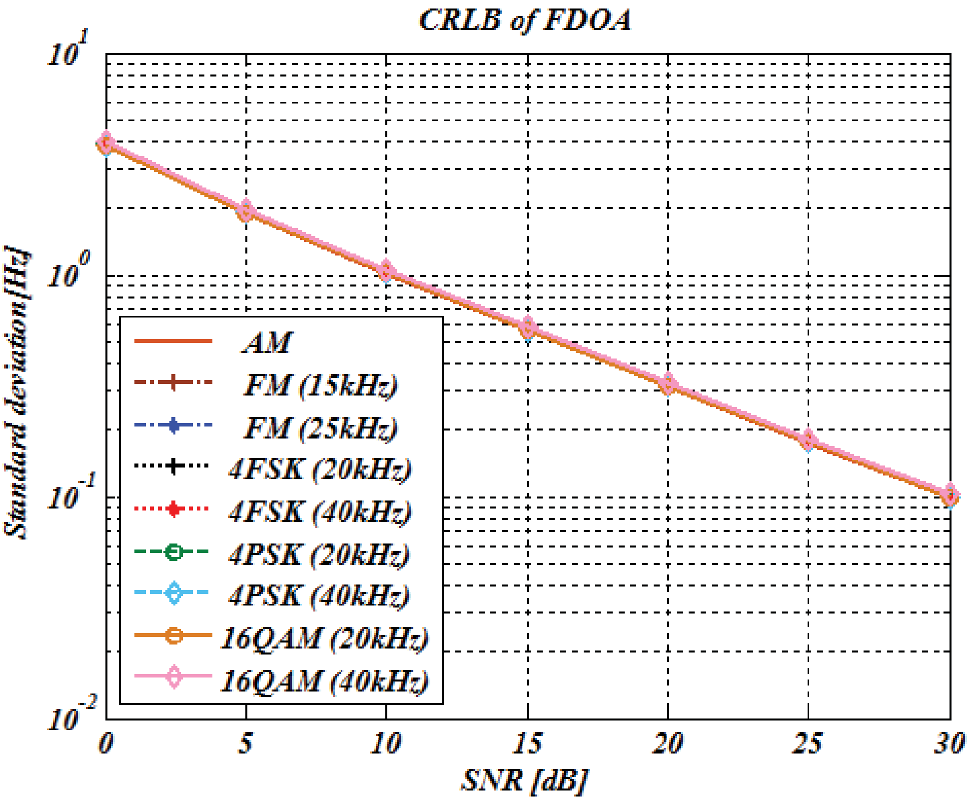

Fig. 2: Cramer-Rao lower bound of FDOA as modulation method and several parameters

PSK and QAM increase. In addition, the spectrum shapes of PSK and QAM are identical, resulting in the same TDOA performance. On the other hand, FDOA performance is the same along with modulation methods as we can see in Figure2 because FDOA performance is affected by the number of samples and the collection time. Modulation methods and main communication parameters do not affect the FDOA performance.

3. Conclusion

In this paper, we analyzed the TDOA/FDOA estimation performance by using CRLB and confirmed our analysis through simulations. From the analysis results we can find that the TDOA performance depends on the spectrum shapes and the FDOA performance is affected only by the collection time. This analysis makes it possible for designers to choose the target performance when SNR values are given as a limiting factor.

4. References

[1] L. R. Paradowski, Microwave emitter position location : present and future, *Proceedings of Microwaves and Radar*, vol4, 1998, pp. 97-116.

[2] D. L. Adamy EW 103: Tactical Battlefield Communications Electronic Warfare, Artech House, 2001.

[3] Y. T. Chan and K. C. Ho, A simple and efficient estimator for hyperbolic location, IEEE transactions on Signal Processing, vol. 42, no. 8, Aug. 1994,

[4] K. C. Ho, An accurate algebraic solution for moving source location using TDOA and FDOA mesurements, *IEEE transactions on Signal Processing*, vol. 52, no. 9, Sep. 2004, pp. 2453-2463

[5] Yeredor and E. Angel, Joint TDOA and FDOA estimation: a conditional bound and its use for optimally weighted localization, *IEEE Transactions on Signal Processing*, vol. 59, no.4, Apr. 2011.

[6] S. Stein, Algorithm for ambiguity function processing, *IEEE transactions on Acoustic Speech and Signal Processing*, vol. ASSP-29, no. 3, Aug. 1981.

[7] P. Panek, Error analysis and bounds in time delay estimation, *IEEE Transactions on Signal Processing*, vol. 55, no. 7, Jul. 2007, pp. 3547-3549.