EECS-1818
Reduction of Searching Range based on Spatial Aliasing for a Grid–Search of ML DOA Estimation

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Abstract

Grid-search based direction-of-arrival (DOA) estimators such as a maximum likelihood method usually require a huge amount of computational complexity. To cope with this problem, an efficient searching range reduction method using spatial aliasing is proposed in this paper. The main idea is to estimate one of the grating lobes only with highly compressed searching range generated by a sparsely deployed linear array and then to eliminate ambiguity on a densely deployed linear array. Performance analysis shows that the computational load in a grid-search can be extremely reduced by the proposed method. In addition, the estimation accuracy is also enhanced due to the enlarged array aperture when using the proposed method.

Keyword: array signal processing; direction of arrival estimation; grid search; spatial aliasing; sparse array

1. Introduction

Direction-of-arrival (DOA) estimation using antenna or sensor array has received considerable interests in the past several decades and has been widely applied for various areas, such as radar, sonar, biomedical engineering, and smart antenna design (H. Krim, 1996). Most of the studies that have been conducted to date on DOA estimation area such as the multiple signal classification (MUSIC) by O. Schmidt (1986) have focused on identifying DOA of closely spaced multiple signals. Among these DOA estimation methods, the grid-search ML method shows asymptotically the best performance in terms of estimation accuracy (Stoica Petre, 1990) even when there are few available snapshots or the signal-to-noise ratio (SNR) is low. However, the grid-search ML method has not attracted array practitioners owing to its high computational complexity even though several efficient methods (Huigang Wang, 2008) have tried to overcome the computational complexity problem.

In this paper, to overcome the problem of computational complexity in the grid-search based DOA estimation method, the combination of a sparse array and a dense array is employed. By using sparse uniform linear array, we intentionally generate spatial aliasing which makes it possible to reduce the computational complexity of grid-search. Spatial aliasing on the sparse array is used to perform full grid search without actual full search by means of the compressed searching range of the grid-search and then the ambiguity induced by the spatial aliasing is eliminated on the dense array. As a result, the amount of computations required for the grid-search based DOA estimation is remarkably decreased. In addition, the enlarged array aperture originating from the use of a sparse array allows a modest degree of performance improvement.

This paper is organized as follows. Section 2 presents brief explanation of grid-search based DOA estimation. The grid search with the compressed searching range, which is the main detail of the proposed method, is described in Section 3. Performance analysis is shown in Section 4 and Section 5 concludes the paper.

2. Grid-Search Based ML Direction of Arrival (DOA) Estimation

Suppose that the D narrowband source signals, $\{s_1(t), s_2(t), \ldots, s_D(t)\}$ from corresponding incident angles $\theta_1, \theta_2, \ldots, \theta_D$, impinge on a uniform linear array (ULA) with M antenna elements indexed by 0, 1, …, m, …, $(M-1)$. The received array signal can be expressed by

$$\mathbf{x}(t) = \mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{n}(t), \quad (1)$$

where $s(t)$ is a signal matrix, $n(t)$ is a noise matrix defined by

$$\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_D(t)]^T, \quad (2)$$

$$\mathbf{n}(t) = [n_0(t), n_1(t), \ldots, n_{M-1}(t)]^T, \quad (3)$$

and array manifold, $\mathbf{A}(\Theta)$, is represented by composition of steering vectors, $\mathbf{a}(\theta)$, as follows

$$\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1)\ \mathbf{a}(\theta_2)\ \ldots\ \mathbf{a}(\theta_D)], \quad (4)$$

$$\mathbf{a}(\theta) = [1,\ e^{-j2\pi\frac{d}{\lambda}\sin\theta}, e^{-2j2\pi\frac{d}{\lambda}\sin\theta}, \ldots,\ e^{-(M-1)j2\pi\frac{d}{\lambda}\sin\theta}]^T \quad (5)$$

Here $[\cdot]^T$ represents the vector transpose, d is the array spacing, and λ is the wavelength of the impinging signal. This study focused only on the angle estimation of the incident signals to simplify the DOA estimation problem with the assumption that the number of signals is known. Thus, the main concern of this paper is to estimate the set of $\Theta=\{\theta_1, \theta_2, \ldots, \theta_D\}$ or $\mathbf{A}(\Theta)=\{\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_D)\}$ from the received signal vector $x(t)$ by using grid-search.

The maximum likelihood (ML) method is a representative grid-search based DOA estimation method. A solution for the ML DOA estimator is obtained by defining a likelihood function and searching for an unknown parameter value that maximizes the

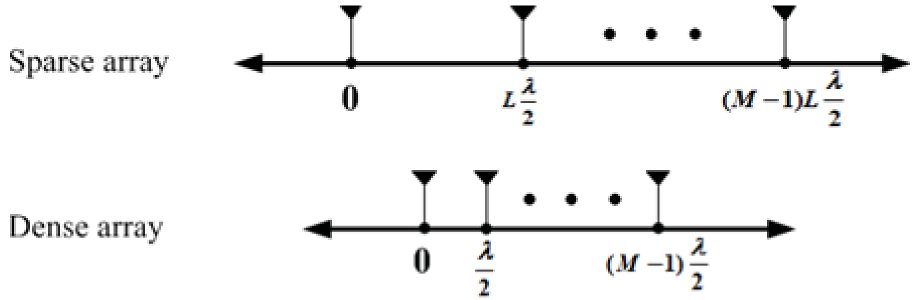


Fig. 1: Array structure for the proposed grid-search range reduction method

likelihood function over all relevant model parameters. The likelihood functions for stochastic signal (SML) and deterministic signal (DML) are respectively as follows :

$$L_{SML}(\Theta) = \log\det\left(\mathbf{P}_{\mathbf{A}(\Theta)}\mathbf{R}_{xx}\mathbf{P}_{\mathbf{A}(\Theta)} - \frac{tr[\mathbf{P}_{\mathbf{A}(\Theta)}\mathbf{R}_{xx}]}{M-D}\mathbf{P}_{\mathbf{A}(\Theta)}\right), \quad (6)$$

$$L_{DML}(\Theta) = \sum_{i=1}^{T} |\mathbf{P}_{\mathbf{A}(\Theta)}\mathbf{x}(t_i)|^2 = tr[\mathbf{P}_{\mathbf{A}(\Theta)}\mathbf{R}_{xx}], \quad (7)$$

where T is the number of snapshots, $\mathbf{R}_{xx}$ is the covariance matrix of $x(t)$, and $tr[\cdot]$ is the trace of a matrix. The projection matrix, $\mathbf{P}_{\mathbf{A}(\Theta)}$, is computed by

$$\mathbf{P}_{\mathbf{A}(\Theta)} = \mathbf{A}(\Theta)(\mathbf{A}^H(\Theta)\mathbf{A}(\Theta))^{-1}\mathbf{A}^H(\Theta), \quad (8)$$

Given a likelihood function, the DOA estimate is obtained by

$$\hat{\Theta} = \max_{\Theta} L(\Theta). \quad (9)$$

To find the solution of (9), the multidimensional search has to be performed over all of candidate angles. Therefore, a huge amount of computational burden is required for a grid-search based ML DOA estimation method.

3. DOA Estimation with Reduced Searching Range Generated by Spatial Aliasing

The antenna array for the proposed efficient grid-search method is composed of a sparse array with $L\lambda/2$ (L>1) spacing and a dense array with $\lambda/2$ spacing as shown in Fig. 1.

The sparse array is used for compressing the grid-search range while the dense array is used for eliminating ambiguity. Focusing on the fact that replica DOAs (θ_{re}) generated from grating lobes and genuine incident DOA (θ_{ge}) produce the same steering vector, finding only arbitrary one of the ambiguous DOAs can provide useful information for predicting other ambiguous DOAs. In other words, grid-search over the entire range of scan angles from −90° to 90° is not necessary on the sparse array.

From the spatial aliasing equation formed by Even, J. (2011), the relation between the genuine incident DOA and the replica DOAs can be derived for some integer k:

$$\frac{d}{\lambda}\sin\theta_{\text{ge}} + k = \frac{d}{\lambda}\sin\theta_{\text{re}}. \qquad (10)$$

Applying (10) to the sparse array with $L\lambda/2$ spacing, the following expressions can be obtained:

$$\frac{L}{2}\sin\theta_{\text{ge}} + k = \frac{L}{2}\sin\theta_{\text{re}}, \qquad (11)$$

and

$$\theta_{\text{re}} = \sin^{-1}(\sin\theta_{\text{ge}} + \frac{2k}{L}). \qquad (12)$$

Because $|\sin\theta| \le 1$, different integer values of k will be given according to $\sin\theta_{\text{ge}}$ as follows:

$$-\frac{L}{2}(1+\sin\theta_{\text{ge}}) < k < \frac{L}{2}(1-\sin\theta_{\text{ge}}). \qquad (13)$$

Hence, the sparse array with L spacing will produce L ambiguous DOAs, which means that it is not necessary to perform a likelihood test over the entire potential DOAs in the searching range unlike in a conventional densely deployed ULA DOA estimator.

The maximum searching angle without spatial aliasing on the sparse array, $\theta_{\text{max},L}$, is determined by using that $d_L = L\lambda/2$ as follows:

$$\theta_{\text{max},L} = \sin^{-1}\frac{\lambda}{2d_L} = \sin^{-1}\left(\frac{1}{L}\right). \qquad (14)$$

As a result, the searching range for grid-search on the sparse array, $\theta_{\text{search},L}$, is compressed over the range given by the following equation:

$$-\sin^{-1}\left(\frac{1}{L}\right) < \theta_{\text{search},L} < \sin^{-1}\left(\frac{1}{L}\right). \qquad (15)$$

From the compressed searching range, the entire DOA estimation procedure using the proposed method is described in Fig. 2. The first step of the proposed method is to perform grid-search with the compressed searching range as (15). With this compressed range, only one estimate for each incident signals is founded.

The second step is composed of obtaining ambiguous DOAs (θ_{am}) from an estimated DOA (θ_{est}) and eliminating ambiguity with searching only for ambiguous DOAs. The ambiguous DOAs from an estimated DOA which is a result of grid search with the compressed searching range on the sparse array is generated by

$$\theta_{am} = \sin^{-1}(\sin\theta_{est} + \frac{2k}{L}), \qquad (16)$$

where k is integer satisfying $|\sin\theta| \le 1$. Because the array spacing of the sparse array is L times larger than the dense array, there exist L ambiguous DOAs including estimated DOA and replica DOAs. The ambiguous DOAs are utilized as the candidate DOAs while avoiding grid-search for entire angle range on the dense array. Therefore, the true DOA is determined on the dense array by searching only for ambiguous DOAs generated from the sparse array.

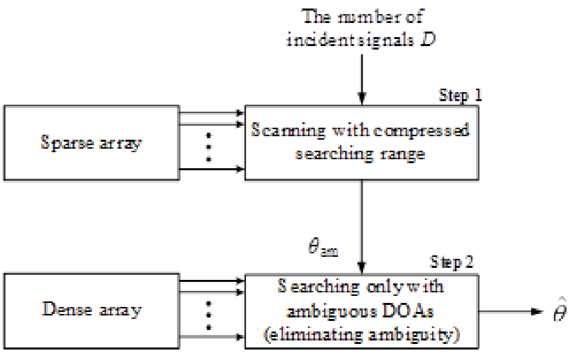


Fig. 2: Procedure of the DOA estimation method with compressed searching range

Table 1: Ratio of the numbers of the required likelihood test for ML DOA estimation of the proposed method compared to the conventional grid search with various number of L and incident signals ($\Delta\theta = 0.1$)

L	**D**			
	1	**2**	**3**	**4**
2	33.44%	11.10%	3.69%	1.23%
3	21.78%	4.66%	1.00%	0.22%
4	16.33%	2.59%	0.41%	0.06%
5	13.11%	1.60%	0.21%	0.02%

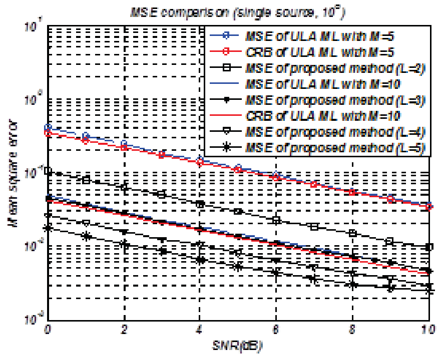
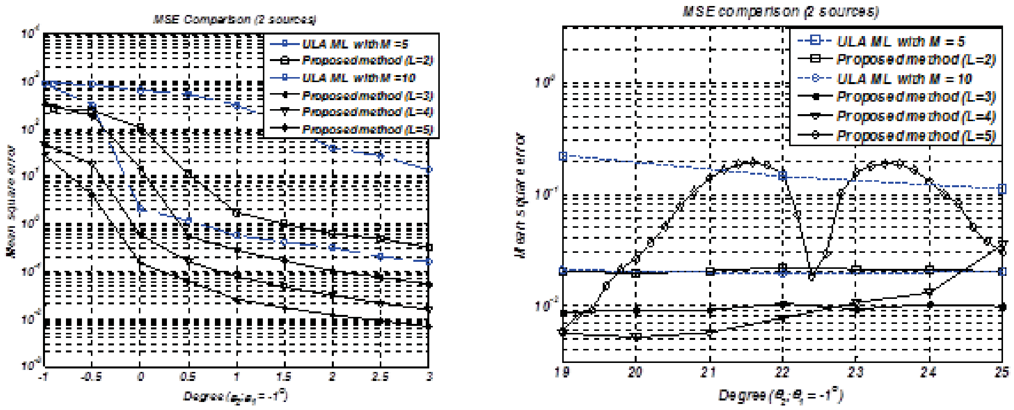


Fig. 3: Comparison of the CRB and mean square errors of ML DOA estimation using the conventional grid-search and the proposed method



(a) (b)

Fig. 4: Comparison of mean square errors of ULA ML DOA estimation using the conventional grid-search with and the proposed method (two source signals, , SNR =10dB, 50 snapshots) (a) angles : θ_1=−1°, θ_2=−1°~3°, (b) θ_1=−1°, θ_2=19°~22°

The number of total required likelihood functions for the ML DOA estimations on the reduced searching range by the proposed method and on the conventional grid-searching range can be easily obtained as

$$\#\text{ of DOAs}_{\text{proposed}} = \underbrace{C\left(\frac{2\sin^{-1}(\frac{1}{L})}{\Delta\theta}, D\right)}_{\text{from the sparse array}} + \underbrace{C(LD,D)}_{\text{from the dense array}}, \quad (17)$$

$$\#\text{ of DOAs}_{\text{conventional}} = C\left(\frac{180}{\Delta\theta}, D\right), \quad (18)$$

where C(n,k) is the k-combinations of n, D is the number of incident signals, L is the parameter for determining array spacing on the sparse array as shown in Fig. 1, and $\Delta\theta$ is the grid size. Ratios of the required likelihood tests for ML DOA estimation using the proposed method compared to the conventional grid-search are listed in Table I according to various number of *Ls* and incident signals (D). Table I shows that the proposed method requires remarkably reduced computational burden compared to the conventional ULA ML method.

The MSE of an ML DOA estimation can be expressed analytically as

$$\mathrm{var}_{\mathrm{ML}}(\hat{\theta}) = \frac{6\sigma^2(MS+\sigma^2)}{S^2TM^2(M^2-1)\left(\frac{2\pi}{\lambda}d\right)^2\cos^2(\theta)}, \qquad (19)$$

where T is the number of snapshots, S and σ^2 is the signal and noise power respectively. Fig. 3 shows the MSEs of ML DOA estimator with the proposed method and the conventional grid-search method using a ULA with M=5 and M=10 as the functions of the signal-to-noise ratio for the measurements of a signal propagated from 10° taken using 50 snapshots. The Cramer-Rao bound (CRB) of the ML DOA estimator using a ULA with M=5 and M=10 is also depicted in Fig. 3. The ML DOA estimation using the proposed grid-search reduction method with L=3,4,5 outperforms ML DOA estimation with the conventional grid-search using a ULA with M=5 due to the enlarged aperture even than the CRB of ML DOA estimation with a ULA while the proposed method with L=2 does not show the better performance than ULA with M=10. Thus, the larger spacing of the proposed method can achieve the better estimation performance.

When multiple sources are considered, the proposed method will provide somewhat variable estimation performance according to the relative position of source signals. Source signals sometimes would seem to be closely spaced owing to spatial aliasing even if they are actually apart from each other. Numerical results are displayed in Figs 4 (a) and (b) over some notable regions for two source signals composed of one fixed source ($\theta_1 = -1^\circ$) and the other source (θ_2) which changes its relative position.

A fixed DOA θ_1=-1° generates ambiguous DOAs for different aperture size of sparse array as follows:

- $L = 5$: $\theta_{\mathrm{am},L}$ = [-54.83°, −24.67°, −1°, 22.49°, 51.49°],
- $L = 4$: $\theta_{\mathrm{am},L}$ = [−31.16°, −1°, 28.85°, 79.28°],
- $L = 3$: $\theta_{\mathrm{am},L}$ = [−43.17°, −1°, 40.48°],
- $L = 2$: $\theta_{\mathrm{am},L}$ = [−1°, 79.28°].

Based on these ambiguous DOAs, two interesting regions were considered to verify the performance of the proposed method. MSE performances are presented for the cases of closely spaced actual DOA in Fig. 4(a) and closely spaced replica DOA in Fig. 4(b). The proposed method estimator with larger L than two outperforms the conventional ULA in terms of spatial resolution in usual cases. However, when the replicas of two sources are closely located each other without regard to its actual relative position, the proposed method experiences performance degradation unlike the ULA ML such as the result of L=5 as shown in Fig. 4 (b). Another interesting phenomenon is that even if two sources are replicas of each other or very closely spaced in the sparse array, they can be also separated by the dense array as shown in Fig. 4 (b) (θ_2= 22.49°). This result comes from the fact that two source signals have the same or very similar ambiguous DOAs in dense array. The above simulation results shows that although the larger L is more likely to experience the closely spaced replica DOAs, the use of the larger L can achieve the higher resolution with the lower computational burden.

5. Conclusion

This paper presented an efficient grid-search method using a sparse array and a dense array. The proposed method could extremely reduce the computational complexity in grid-search based DOA estimators and makes them to be implementable without degradation of searching performance. In addition, the combination of a sparse array and a dense array can achieve the higher estimation accuracy due to its enlarged array aperture compared to the conventional grid-search method using a ULA. Therefore, the proposed method can be effectively used in grid-search based estimations.

6. Acknowledgment

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A2008555).

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