

PERFORMANCE ANALYSIS OF TDOA/FDOA ESTIMATION FOR FM COMMUNICATION SIGNALS

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Abstract- A study on emitter localization using time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements has been increased recently. A TDOA/FDOA system generally consists of two parts: TDOA/FDOA estimation from the unknown received signal (extraction part) and position estimation of the emitter (localization part). Thus, it is important to accurately estimate TDOA and FDOA prior to the localization part. However, in electronic warfare circumstance, due to lack of information of the received signal, the accuracy may vary with chosen parameters and a structure of the receiver. In this paper, we consider the case that an enemy's emitter transmits an FM-modulated communication signal and phased-locked-loop is no longer available in the receiver because the modulation method is also unknown. To obtain the theoretical limit of the estimation performance in this situation, we analyze the estimation performance of TDOA/FDOA from the received signals corresponding to the bandwidth of the noise reduction filter which is one of the design parameter of the receiver by using Cramer-Rao lower bound (CRLB) and simulations.

Keywords- Time Difference of Arrival (TDOA), Frequency Difference of Arrival (FDOA), Electronic Warfare System, Cramer-Rao Lower Bound (CRLB)

I. INTRODUCTION

TDOA/FDOA-based emitter localization methods have drawn much interest due to their higher performance in the electronic warfare system. A TDOA/FDOA-based localization generally consists of two steps. In the first stage, TDOA and FDOA information are extracted from the received unknown signals at several sensors (extraction part). Sequentially, localization is performed by using the TDOA and the FDOA are extracted in the first stage (localization part). Thus, the final localization performance of the TDOA/FDOA system is affected by the accuracy of the estimated TDOA/FDOA values. Consequently, to analyze the final localization performance, the analysis on the accuracy of the TDOA/FDOA estimates should be preceded.

However, in electronic warfare circumstance, preliminary information about an emitter's signal, such as modulation method and signal bandwidth, is completely unknown. This lack of knowledge causes a difficulty in selecting design parameters of the receiver, such as collection time and bandwidth of the noise reduction filter. Especially, in the case that the modulation method is FM, it is impossible to demodulate by using a phased-locked loop while in the other modulation methods, such as amplitude modulation, frequency-shift keying, phase-shift keying, and quadrature-amplitude modulation, the received signal can be demodulated by baseband down conversion. So, in the FM modulated signal we cannot help using just down conversion like in the other modulation methods for demodulation. Due to this difficulty in demodulating FM-modulated signals, it is meaningful to analyze the performance of the FM signal demodulated by just down conversion. In this

respect, we analyze the estimation performance along with frequency deviation and bandwidth of the noise reduction filter for TDOA/FDOA which are extracted from unknown FM-modulated signals.

To achieve this goal, we derive the Cramer-Rao lower bound for the TDOA and FDOA estimates of the baseband signal which is acquired from a down-converted FM signal. Also, we analyze the estimation performance for TDOA/FDOA of FM communication signals along with bandwidth of the noise reduction filter by using the root-mean square error for the corresponding simulation results.

II. CRAMER-RAO LOWER BOUND FOR FM SIGNALS AND SIMULATION RESULTS

2.1 Cramer-Rao lower bound for FM signals

Estimation accuracy for TDOA and FDOA measurements of two received signals can be analyzed using CRLB theoretically. Thus, in this subsection, we explain the Cramer-Rao lower bound (CRLB) for TDOA and FDOA measurements based on FM signals.

FM signal which propagated from unknown emitter is as follow:

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi f_\Delta \int_0^t u(t') dt'\right) \quad (1)$$

Where A_c is amplitude, f_c is carrier frequency, $u(t)$ is unknown message signal and f_Δ is frequency deviation. To demodulate the FM signals, phase-locked loop is generally used. However, although the received signal is modulated on FM, we have to only carry out down conversion from the radio frequency

to the baseband for versatile use to adjust amplitude modulation, phase shift modulation and quadrature amplitude modulation because modulation method of the received signal is completely unknown in the electronic warfare system. Consequently, baseband signal which is converted from (1) is acquired as follow:

$$s_b(t) = A_c \cos 2\pi f_\Delta \int_0^t u(t') dt' \quad (2)$$

Thus, the baseband received signal depends on frequency deviation f_Δ and message signal.

To estimate the TDOA/FDOA information, two sensors have to be located far from each other and intercept the unknown signal which is propagated from the emitter. These two intercepted signal at each sensor defined by

$$\begin{aligned} s_{b1}(t) &= A_c e^{j2\pi v_1 t} \cos 2\pi f_\Delta \int_{\tau_1}^t u(t') dt' + n_1(t) \\ s_{b2}(t) &= A_c e^{j2\pi v_2 t} \cos 2\pi f_\Delta \int_{\tau_2}^t u(t') dt' + n_2(t) \end{aligned} \quad (3)$$

where τ_1 and τ_2 are time delays from the unknown emitter to each sensor, respectively. v_1 and v_2 are Doppler frequencies and $n_1(t)$ and $n_2(t)$ are additive white Gaussian noise at each sensor. TDOA and FDOA are defined respectively as follows:

$$\begin{aligned} \tau_d &= \tau_1 - \tau_2 \\ v_d &= v_1 - v_2 \end{aligned} \quad (4)$$

Let $\hat{\tau}_d$ and \hat{v}_d be the estimates of TDOA and FDOA. Since the CRLB represents a lower bound on the variance of the estimate, theoretical performance of any unbiased estimator for TDOA and FDOA can be derived using CRLB, which was derived in [5] and [6], as follows:

$$\begin{aligned} \text{var}(\hat{\tau}_d) &\geq \frac{1}{4\pi^2 B T \gamma B_{rms}^2} \\ \text{var}(\hat{v}_d) &\geq \frac{1}{4\pi^2 B T \gamma T_{rms}^2} \end{aligned} \quad (5)$$

Where B is noise bandwidth, T is collection time, B_{rms} is root-mean-square (RMS) bandwidth and T_{rms} is root-mean-square (RMS) collection time, defined as follows:

$$\begin{aligned} B_{rms} &= \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |S_{b1}(f)|^2 df}{\int_{-\infty}^{\infty} |S_{b1}(f)|^2 df}} \\ T_{rms} &= \sqrt{\frac{\int_0^T t^2 |s_{b1}(t)|^2 dt}{\int_0^T |s_{b1}(t)|^2 dt}} \end{aligned} \quad (6)$$

where $s_{b1}(t)$ can be replaced $s_{b2}(t)$. Also, γ is signal-to-noise ratio (SNR) defined by

$$\frac{1}{\gamma} = \frac{1}{2} \left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_1 \gamma_2} \right] \quad (7)$$

where γ_1 and γ_2 are SNRs at sensor1 and sensor2, respectively.

Consequently, from (3) and (6), we can find that the performance for TDOA depends on RMS bandwidth. It means that frequency deviation is major factor to affect the estimation performance. Otherwise, the performance for FDOA depends on collection time.

2.2 Simulation Results

In this subsection, to analyze the estimation performance for TDOA, we carried out the simulation for TDOA estimates according to frequency deviation and bandwidth of the noise reduction filter. Figure 1 and Figure 2 show magnitude of baseband FM signal which is applied by (2) on frequency domain when the frequency deviation are 15 kHz and 75 kHz respectively. In these figures, rectangular shape represent the region of the noise reduction filter. As shown in the Figure 1, the signal is preserved after that the signal pass through the noise reduction filter. However, in Figure 2, we encounter the signal loss which is caused by noise reduction filter. So, although the theoretical estimation performance for TDOA is improved as frequency deviation increases, the final performance is worsened when the bandwidth is narrow. Otherwise, if the bandwidth of the noise reduction filter is wide, noise power increases. Therefore, it is necessary to analyze the effect to the estimation performance as the bandwidth of the noise reduction filter.

Figure 3 shows the estimation performance versus signal to noise ratio (SNR) along with frequency deviation when the bandwidth of the noise reduction filter is 20 kHz (-10 kHz ~ 10 kHz). CRLB is represented only when the frequency deviation is 5 kHz for readability of the figures. Also, we use Stein's algorithm [4] and calculate the SNR without signal loss caused by noise reduction filter. That is, the SNR in the figure represent the input SNR before the signal pass through the noise reduction filter. When the frequency deviation of FM signals are 15 kHz, 35 kHz and 75 kHz, the estimation performance are worse than the frequency deviation is 5 kHz in Figure 3. It means that much loss of the signal is occurred by the noise reduction filter. Under the similar way, Figure 4 shows the estimation performance when the bandwidth of the noise reduction filter is 60 kHz (-30 kHz ~ 30 kHz). In contrast with the above case (bandwidth: 20 kHz), the estimation performance for TDOA is generally

improved as frequency deviation increases likewise the theoretical analysis. However, the estimation performance are similar or worse when the frequency deviation is 75 kHz compared to 35 kHz. Lastly, Figure 5 shows the performance when the bandwidth is 100 kHz (-50 kHz~ 50 kHz). As shown the figure, although signal loss is reduced, tendency of the estimation performance is still similar when the bandwidth is 60 kHz. Therefore, 60 kHz is more appropriate than 20 kHz and 100 kHz for the bandwidth of the noise reduction filter considering that an increase of the noise power when the bandwidth is 100 kHz.

CONCLUSION

In this paper, we analyzed the TDOA/FDOA estimation performance by using CRLB and root-mean-square error of TDOA estimate through our simulations when the unknown received signal is FM-modulated communication signal.

From the theoretical analysis, we found that the estimation performance for TDOA is affected by the collection time and the frequency deviation; and the performance for FDOA is related to the collection

time. Also, we presented the appropriate bandwidth of the noise reduction filter based on the simulation results of showing the relation between the frequency deviation and the bandwidth. The analysis results are expected to be useful to determine the design parameter for the electronic warfare receiver.

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