

# A GENERALIZED SPATIAL DIFFERENCING METHOD FOR DIRECTION-OF-ARRIVAL ESTIMATION OF UNCORRELATED AND COHERENT SOURCES

<sup>1</sup>GEUN-HO PARK, <sup>2</sup>YEOUNG-GWANG SEO, <sup>3</sup>JONG-WOO SHIN, <sup>4</sup>HYOUNG-NAM KIM

<sup>1,2,3,4</sup>Pusan National University  
E-mail: 4 hnkim@pusan.ac.kr

**Abstract-** Direction-of-arrival (DOA) estimation algorithms such as subspace based techniques have been extensively studied over the past few decades. However, these algorithms are not applicable to DOA estimation for coherent signals (i.e., fully correlated). As an attempt to remove the correlation between incident signals, a spatial differencing method (SDM) was proposed under the condition that both the uncorrelated and the coherent signals exist. By taking the advantage of the property that the information of the coherent signals can be extracted from the received data, SDM could increase the maximum number of resolvable sources. In several cases, however, the conventional SDM may fail to detect coherent signals because the conventional spatial differencing matrix is derived from the part of the covariance matrix. In this paper, a generalized spatial differencing method is proposed for DOA estimation of coherent signals by using the entire spatial differencing matrices. Through the simulations, the proposed method is confirmed that it resolve more coherent sources and also improve the accuracy of the DOA estimates.

**Keywords-** DOA Estimation, Uncorrelated Signals, Coherent Signals, Spatial Smoothing, Spatial Differencing Method, Uniform Linear Array.

## I. INTRODUCTION

Direction-of-arrival (DOA) estimation algorithms have been studied over the past few decades. Subspace based techniques, such as multiple signal classification (MUSIC) and minimum-norm (MN), have been proposed to estimate the incident angles of narrowband sources. However, there is a problem that coherent sources, caused by multi-path environment, critically degrade the performance of these algorithms.

To solve this problem, several preprocessing schemes have been developed. First, a spatial smoothing scheme was proposed to decorrelate the incident signals. Spatial smoothing can resolve the coherent signals by constructing a spatially smoothed covariance matrix. Second, the required number of antennas can be further decreased by using the enhanced techniques referred to as the forward/backward spatial smoothing (FBSS).

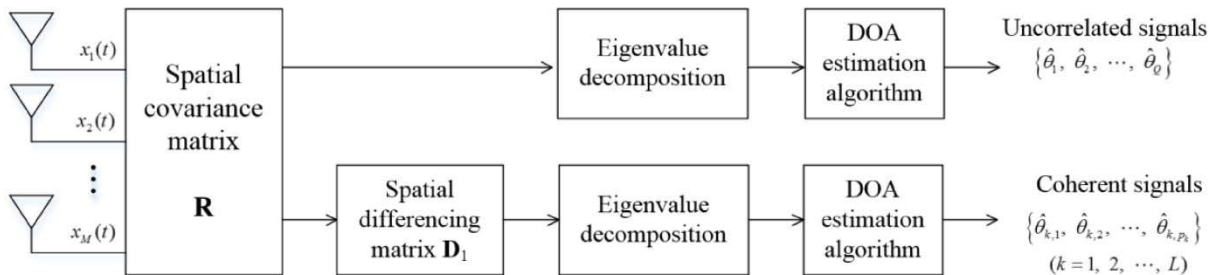
Finally the improved spatial smoothing (ISS) increased the accuracy of DOA estimations, using cross-correlation terms of a spatial covariance matrix.

To resolve more signals than the number of array elements, spatial differencing method (SDM) was proposed when the uncorrelated and the coherent sources simultaneously impinge on the uniform-linear array. By exploiting the property that the information of uncorrelated sources can be removed, DOA estimation for coherent signals can be effectively conducted.

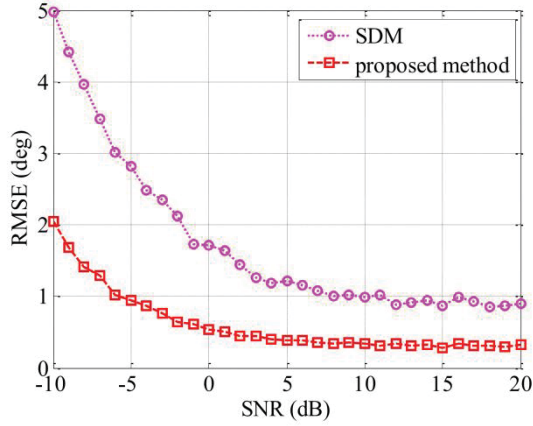
As a result, SDM can increase the maximum number of resolvable sources. However, the conventional SDM technique may fail to estimate DOAs of the coherent sources because SDM uses only the part of the covariance matrix. In this paper, we present a new spatial differencing method by defining a new spatial differencing matrix where whole information of a covariance matrix is used. By doing so, the proposed method increase the accuracy of the DOA estimates and the detectable number of coherent signals.

## II. SPATIAL DIFFERENCING METHOD

As an attempt to increase the number of detectable signals, SDM has successfully resolved



**Figure 1: Block diagram of the spatial differencing method.**



**Figure 2: RMSE curves of the DOA estimates versus input SNR.**

more coherent and uncorrelated sources than the number of antennas. As shown in Fig. 1, DOAs of uncorrelated signals are estimated from the conventional covariance matrix at first. The coherent sources can be resolved by using a spatial differencing matrix, which contains only the information of DOAs for coherent signals. Exploiting the property that the covariance matrix of the uncorrelated signals is a to eplitz matrix, a spatial differencing matrix  $\mathbf{D}_1$  is obtained by

$$\mathbf{D}_1 = \frac{1}{N} \sum_{m=1}^N (\mathbf{R}_1 - \mathbf{J}_{M-N+1} \mathbf{R}_m^* \mathbf{J}_{M-N+1}), \quad (1)$$

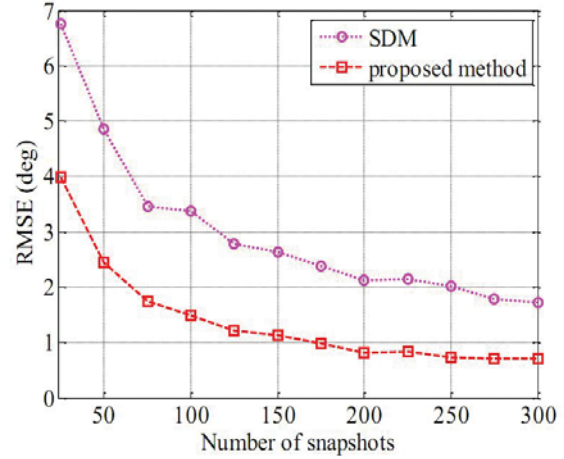
where  $M$  is the number of antennas,  $N$  is the number of subarrays.  $\mathbf{R}_m$  and  $\mathbf{J}_m$  denote the covariance matrix of  $m$ th subarray and the exchange matrix with ones on its anti-diagonal and zeros elsewhere. The superscript  $(\cdot)^*$  stands for the complex conjugation without transposition. DOAs for coherent signals are calculated by applying the DOA estimation algorithms to the spatial differencing matrix  $\mathbf{D}_1$ .

### III. A PROPOSED METHOD

The conventional SDM extracts the information of coherent DOAs based on  $\mathbf{R}_1$  (i.e., first sub-covariance matrix). However, additional spatial differencing matrices can be derived based on other sub-covariance matrices. For example, spatial differencing matrix  $\mathbf{D}_3$  is computed from the covariance matrix of 3rd subarray.

In this paper, we derive the  $r$ th spatial differencing matrix  $\mathbf{D}_r$  ( $r = 1, \dots, N$ ) and justify the generalized spatial differencing matrix which makes best use of the entire information of the received data.

The  $r$ th spatial differencing matrix  $\mathbf{D}_r$  and the generalized spatial differencing matrix  $\mathbf{D}$  are defined by



**Figure 3: RMSE curves of the DOA estimates versus the number of snapshots.**

$$\mathbf{D}_r = \frac{1}{N} \sum_{m=1}^N (\mathbf{R}_r - \mathbf{J}_{M-N+1} \mathbf{R}_m^* \mathbf{J}_{M-N+1}), \quad (2)$$

$$r = 1, \dots, N,$$

$$\mathbf{D} = \sum_{r=1}^N \mathbf{D}_r^2. \quad (3)$$

DOA estimations of the coherent sources are calculated by eigen-decomposition of  $\mathbf{D}$  and DOA estimation algorithms. Estimation performance of the proposed method can be confirmed by simulations in the next section.

### IV. SIMULATION RESULT

In this section, several simulations are conducted to evaluate the proposed technique. For all simulations, the conventional SDM and the proposed SDM are carried out for comparison.

At first, the root-mean-square-error (RMSE) of the DOA estimates is compared by using root-MUSIC algorithm.

The number of uniform linear array elements is taken as  $M=9$  with inter-element spacing  $d = \lambda/2$ . Assume that two uncorrelated signals come from  $\{-50^\circ, -30^\circ\}$  and two groups of coherent sources come from  $\{0, 10\}$  and  $\{-30^\circ, -20^\circ, -60^\circ\}$ .

The RMSE of the DOA estimates versus the SNR and the number of snapshots are shown in Figs. 2 and 3. In each figure, the number of snapshots is taken as  $K=1000$  and the SNR is taken as 10 dB.

These figures show that the proposed SDM technique estimates the DOAs of coherent sources more accurately as the number of snapshots and the SNR increase.

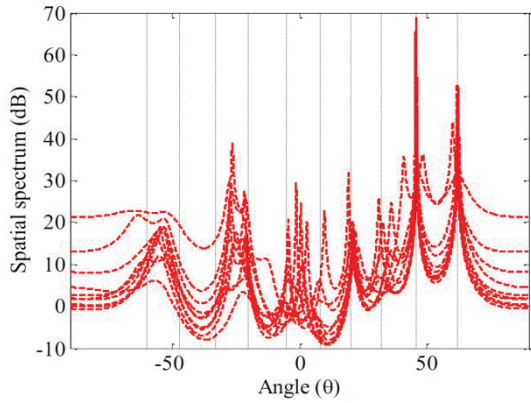


Figure 4: Ten independent runs of MUSIC spectrum using the conventional SDM.

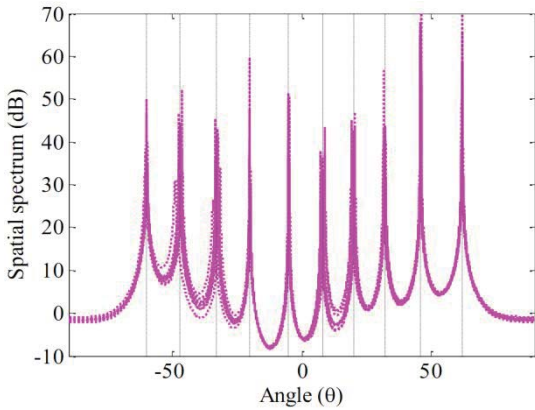


Figure 5: Ten independent runs of MUSIC spectrum using the proposed method.

In the second simulation, we compare the resolvable number of incident signals. The number of sensors is taken as  $M = 12$  and we assume that the three uncorrelated sources and ten coherent sources come to the uniform linear array. Ten independent runs using MUSIC spectrums of the conventional and the proposed method are shown in Figs. 4 and 5. From these figures, we confirm that the proposed method can resolve more sources than the conventional SDM technique.

## CONCLUSION

A spatial differencing method has been studied for resolving DOAs of the coherent and the uncorrelated sources more than the number of array elements. In several cases, however, the conventional SDM failed to detect coherent signals because the conventional spatial differencing matrix was derived from the part of the spatial covariance matrix. In this paper, we proposed a generalized spatial differencing method for DOA estimation of the coherent signals by using the entire spatial differencing matrices. The proposed method does not only improve the accuracy of the DOA estimates but also increases the number of detectable coherent sources. Accordingly, it is expected to be realized more effectively in various applications such as radar, sonar, and smart antenna systems.

## REFERENCES

- [1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. AP*, vol. 34, no. 3, pp. 276-280, Mar. 1986.
- [2] R. Kumaresan, D. W. Tufts, "Estimating the angles of arrival of multiple plane waves," *IEEE Trans. Aerosp. Elect. Systems*, vol. AES-19, pp. 134-139, 1983.
- [3] T. J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acoust. Speech Signal Process*, vol. 33, no. 4, pp. 806-811, Aug. 1985.
- [4] S. U. Pillai and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Trans. Acoust. Speech Signal Process*, vol. 7, no. 1, pp. 8-15, Jan. 1989.
- [5] W. Du and R. L. Kirlin, "Improved spatial smoothing techniques for DOA estimation of coherent signals," *IEEE Trans. Signal Process*, vol. 39, no. 5, pp. 1208-1210, May. 1991.
- [6] F. Liu, J. Wang, C. Sun and R. Du, "Spatial differencing method for DOA estimation under the coexistence of both uncorrelated and coherent signals," *IEEE Trans. Antenna and Propagation*, vol. 60, no. 4, pp. 2052-2062, April. 2012.

★★★